

The Multi-tanh Principle: A Tutorial Overview

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Abstract—This paper reviews a class of linear transconductance cells, having proven value in a variety of communications applications, characterized by the use of parallel- or series-connected sets of differential pairs of bipolar transistors whose inputs and outputs are connected in parallel. These cells invoke a well-developed concept, known as the “multi-tanh principle.” The key idea is that the individually nonlinear (hyperbolic tangent, or *tanh*) transconductance functions may be separated along the input-voltage axis to achieve a much more linear overall function. The simplest of these is called the “doublet”; the linearity criterion and noise behavior are discussed in detail. Some novel forms are presented. Higher order cells, including the “triplet,” are then discussed, together with a novel method for achieving linear-in-dB gain control with an important modification for extending the dynamic range.

Index Terms—Active mixers, linearization, transducers, V-I conversion, variable-gain cells.

I. INTRODUCTION

DURING the past 25 years, a particular class of bipolar cells based on the common differential bipolar pair has undergone a transformation from an academic curiosity, used in short courses as an interesting example of how one can shape the transconductance behavior of such cells to address a variety of hypothetical requirements, to a valuable contemporary concept which appears to be enjoying something of a renaissance. Since these cells depend on combining a multiplicity of offset hyperbolic tangent, or *tanh*, functions [1], the name “multi-tanh” was coined by the author to describe the topological and mathematical aspects of the concept. Though awkward, the term has nevertheless gained acceptance. This paper provides a unified description of the principles and cell realizations, demonstrates their value in high-performance analog-signal processing applications, elucidates some little-known pitfalls, and describes for the first time some useful variants and extensions.

The multi-tanh concept is a technique for extending the voltage capacity of a transconductance (g_m) cell, or an amplifier, mixer, continuous-time filter, or other active element based on such a cell, by using at least two, and in general N , differential pairs operating in parallel, each having a base offset voltage applied by some means, which splits the individual g_m functions along the input-voltage axis. This allows the cell to handle larger voltage swings at its input, while the overall transconductance is more linear, providing a low-distortion function. The earliest and simplest form, having $N = 2$ and called the doublet, dates to 1968. It was reported by Baldwin and Rigby [2], although the emphasis of that work was on drift compensation in IC amplifiers rather

than linearity improvement. Various test cells were fabricated in a series of experimental project chips fabricated by the author at Tektronix during the years 1968–1972. The National Semiconductor LM121 preamplifier, introduced in 1972, used the doublet, but again with an emphasis on lowering input drift, challenging chopper-stabilized amplifiers in this regard [3].

The value of doublets in addressing open-loop distortion and slew rate limitations in operational amplifiers was delineated by the author in several Analog Devices memoranda in the mid-1970's and also recognized by Schmook [4], with the objective presented in terms of “transconductance reduction.” During the period 1975 to the present, the multi-tanh concept has been widely taught in short courses and adopted as a thesis topic based on material provided by the author (for example, Andersen in 1978 [5], Mack in 1979 [6], and Gold in 1988 [7]) and discussed in at least one book [8]. Recently, these vintage ideas have found use in quadrature voltage controlled oscillators (VCO's) (for example, by Brown [9], in several commercial IC's¹), in tunable filters (for example, by Voorman [10] and Tanimoto [11]), and in miscellaneous nonlinear applications such as the squaring-function cells described by Kimura [12]. Surprisingly, in view of the very extensive prior art in the public domain, several multi-tanh patents have been issued in recent years [13]–[16]. The extension to MOS implementations operating in weak inversion is obvious and straightforward; the scaling effects resulting from the altered coefficient of kT/q and errors arising from back-gate bias need consideration.

Section II first reviews the importance of providing highly linear transconductance and discusses the basic metrics of high dynamic range systems. The term “harmonic signature” is introduced, and its value explained. Section III outlines the general case and states the principle in its broadest terms. The notion of an “elastic transconductance” is explained, in reference to a special class of multi-tanh cells whose large-signal transfer characteristic can adapt to varying signal-amplitude requirements in a wide-dynamic-range application.

While of considerable academic interest, generalized high-order multi-tanh cells remain of limited practical value. Accordingly, Section IV analyzes the elegant and eminently practical multi-tanh doublet in depth. This simple cell immediately provides a dramatic improvement in linearity compared to a single differential pair, having a theoretical noise penalty of only 2 dB, while preserving the useful property of a transconductance which is a linear function of the bias currents. The doublet has been widely used in fixed- and variable-gain mixers, IF amplifiers, tunable filters, and demodulators in Analog Devices communications products. Alternative methods for

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¹For example, the Analog Devices AD6432 uses doublets in a wide-tuning-range quadrature VCO which forms part of an I/Q demodulator.

introducing the offsets in the tanh functions are described, including an ultra-linear fixed- g_m variant. Series-connected variants are also presented.

Section V takes the next natural step to the multi-tanh triplet. This cell uses three pairs of transistors, the outer two of which are now offset by a larger amount than for the doublet, with an attendant increase in the signal capacity. In its basic form, the input noise is greater than for the doublet. A novel biasing/gain-control method is later presented which provides precise, truly exponential (that is, linear-in-dB) gain-control while simultaneously lowering the noise penalty compared to a fixed-configuration triplet. This so-called “tripus” cell implements the elastic transconductance concept in an elegant and efficient manner.

The emphasis throughout this paper is necessarily focused on fundamental aspects of cell behavior, with only a brief mention of certain practical concerns, such as the degradation in noise caused by the biasing methods, noise contributions of ohmic resistances in the transistors, the effect of mismatches, and so on. Many other practical matters affect the utility of multi-tanh cells, such as their behavior at very high frequencies, which, for reasons of space, have been omitted. The primary purpose of the present work is to bring together a number of related ideas into a unified treatment of the underlying principles and promote their wider utilization.

II. DYNAMIC RANGE CONSIDERATIONS

An important objective of analog signal-processing cells in communications applications, for which the multi-tanh principle is well-suited, is to combine *low distortion* with *low noise* in order to achieve a *high dynamic range*. This can be defined as the ratio of the maximum signal-handling capacity to the noise floor. The former may be defined (by arbitrary convention) as that sinusoidal input amplitude for which the cell output is 1 dB below the ideal (linear-response) value, the so-called 1-dB compression point. This is often expressed as a power, using the variable $P_{1\text{dB}}$, in decibels above 1 mW, or dBm.² However, the majority of integrated-circuit cells are *not* fundamentally power-responding, but limited by *voltage* constraints. Accordingly, we will here express signal capacity as a *voltage* amplitude $V_{1\text{dB}}$, measured in dBV, noting in passing that 0 dBV corresponds to +10 dBm in 50 Ω .

The noise floor for an IC cell is fairly completely defined by the net voltage-noise spectral density, expressed in nV/ $\sqrt{\text{Hz}}$, referred to the cell input when driven from a specified source impedance. In the case of multi-tanh cells, the contribution of the input current noise will usually be relatively small, and thus the short-circuit noise spectral-density provides a good measure of noise performance. The dynamic range can be stated for a 1-Hz noise bandwidth, as dBc-Hz. For example, in a system having a 1-dB gain-compression amplitude ($V_{1\text{dB}}$) of 200 mV and a voltage-noise spectral density of 2 nV/ $\sqrt{\text{Hz}}$, the dynamic range would be 160 dBc-Hz. In applications where the noise bandwidth is known, the dynamic range is expressed

simply in decibels. For this example, it would be 107 dB in a 200-kHz bandwidth.

While this is a simple and useful idea, there are many situations in which another factor needs to be taken into account in assessing dynamic range and this is the third-harmonic distortion, or HD3. This is present at input levels well below $V_{1\text{dB}}$ and is invariably of more interest than any other order of distortion, since it gives rise to *intermodulation side-tones*, when the cell is excited by two sinusoidal carriers of frequency ω_1 and ω_2 (generally corresponding to two received signals), which fall on either side of these carriers at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$. These spurious signals are thus indistinguishable from genuine signals in adjacent channels. Thus, the emphasis in discussing the distortion of multi-tanh cells will be largely on HD3.

The standard treatment of intermodulation effects in a system of chained cells assumes that the prevailing nonlinearity of each cell has a simple compressive cubic form, $V_{\text{OUT}} = V_{\text{IN}} - V_{\text{IN}}^3/V_Z^2$, where V_Z determines the magnitude of the nonlinearity. Substituting $E \sin \omega t$ for V_{IN} shows that a third-harmonic component V_{H3} of magnitude $(E^3/4V_Z^2) \sin 3\omega t$ is generated, which increases as the cube of the amplitude E , that is, on a graphical slope of three, using decibel axes for the input and output amplitudes. This simplification permits the use of the concept of an *intermodulation intercept*, the point at which the third-harmonic line, extrapolated from a point corresponding to a low-level test signal, meets the extrapolated fundamental line, usually measured along the input axis. This occurs at $V_{\text{IN}} = 2V_Z$. The variable V_{IH3} is here used to denote the *single-tone third-harmonic intercept voltage*. The two-tone intercept occurs at a level which is $\sqrt{3}$ or 4.77 dB below V_{IH3} .

This metric is supposed to adequately define the quality of a signal-processing cell in regard to the generation of spurious responses, and spreadsheet analyzes of a complete system (signal chain) assume this cubic formulation. Unfortunately, the more complicated distortion characteristics of many contemporary cells, in particular, the multi-tanh cells to be described, casts doubt on its value. This is because the nonlinearity does *not* follow a simple cubic law for inputs of moderate amplitude, as we shall see. It follows that the third-harmonic distortion has complex character, and the notion of *intercept* becomes less certain.

We will thus fall back on the idea of the harmonic signature [8] to explore the distortion behavior of various multi-tanh cells. This is the familiar plot of the fundamental, of relative decibel magnitude H_1 , and one or more of the harmonics, particularly the third harmonic component H_3 , in response to a sinusoidal excitation of increasing amplitude. The third-harmonic distortion for a given excitation amplitude is simply the decibel difference $H_3 - H_1$ and is denoted by HD3. The term “signature” provides a useful reminder that every circuit (and bias condition) exhibits a unique form, as the examples presented here clearly demonstrate.

III. THE GENERAL CASE

The incremental transconductance (g_m) of a bipolar junction transistor (BJT) differential pair varies considerably with the

² A power level of 0 dBm is 1 mW. In RF applications a load impedance of 50 Ω is assumed. In this case, 0 dBm corresponds to a sinewave voltage amplitude of 316.2 mV.

instantaneous input voltage applied across the bases. The functional form of its dc transfer function is well known

$$I_{OUT} = I_E \tanh \frac{V_{IN}}{2V_T} \quad (1)$$

where I_{OUT} is the differential output current, I_E is the emitter bias ("tail") current, and V_T has the usual meaning of kT/q (25.85 mV at $T = 300$ K). From this it follows that the incremental transconductance varies with V_{IN}

$$g_m(V_{IN}) = \frac{dI_{OUT}}{dV_{IN}} = \frac{I_E}{2V_T} \operatorname{sech}^2 \frac{V_{IN}}{2V_T}. \quad (2)$$

This function limits the maximum signal that can be applied before unacceptable distortion occurs. The g_m is reduced to 50% of its peak value g_{mo} (at $V_{IN} = 0$) for $V_{IN} = 45.6$ mVP, where this notation denotes a voltage (or signal-voltage capacity) that is proportional to absolute temperature, with an implied normalizing temperature of 300 K. The simple differential pair generates an HD3 of about 1% for a sinusoidal excitation of only 18 mVP in amplitude, independent of the bias current. It should be noted that all cells based on BJT differential pairs exhibit this fundamental variation in signal capacity over temperature, and that a worst-case performance assessment should be made at the lowest operating temperature. Thus, the input amplitude for an HD3 of 1% is lowered to 13 mV at -55°C (that is, $18 \times 218/300$). In many applications of g_m cells, for example, mixers and IF amplifiers, much larger signals must be handled, often due to the presence of strong interferers or blocking signals, and the critical requirement for low intermodulation products.

On the other hand, the fundamental noise floor of this basic cell for a tail current of 1 mA, determined by shot noise mechanisms, is very acceptable, being roughly $0.93 \text{ nV}/\sqrt{\text{Hz}}$ at $T = 300$ K, about the same as the Johnson noise of a $50\text{-}\Omega$ resistor. Classical resistive emitter degeneration is valuable for improving linearity, but its use incurs a significant noise penalty. Furthermore, cells using degeneration are not as amenable to accurate gain control through bias current variation as those which depend directly on a linear $g_m - I_C$ relationship, the fundamental translinear behavior of the bipolar junction transistor [17].

The fully general case for the multi-tanh principle was originally shown in a quite different context, that of sine function synthesis [1], in 1977. Fig. 1 shows the general topology. The N differential pairs are offset along the voltage axis, and each has its own current source. This circuit synthesizes the function

$$I_{OUT} = \sum_{j=1}^N I_j \tanh \left\{ \frac{V_{IN} + V_j}{2V_T} \right\} \quad (3)$$

where I_j is the tail current to the j th stage and V_j is the base offset voltage associated with that stage. We have complete freedom to choose I_j and V_j , though for very large N the tail currents will usually be equal, and the offsets spaced uniformly; some shaping of these coefficients can improve the linearity at the extremities of the range. The total g_m of

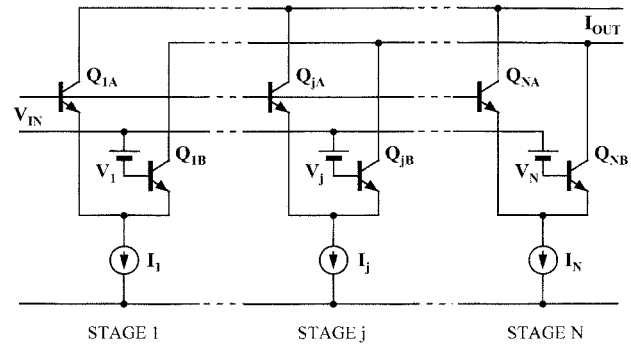


Fig. 1. A generalized multi-tanh system.

these N stages is

$$g_m = \sum_{j=1}^N \frac{I_j}{2V_T} \operatorname{sech}^2 \left\{ \frac{V_{IN} + V_j}{2V_T} \right\}. \quad (4)$$

There are several ways in which the necessary offsets can be introduced. For low-order cells ($N = 2$ for the doublet and $N = 3$ for the triplet) they can be most simply generated using emitter-area ratios. It should be noted that these automatically generate the proportionality to absolute temperature (PTAT) required for correct operation of the cell. For large values of N , they can be introduced using PTAT currents operating on one or two chains of resistors. Fig. 2 shows an illustrative scheme augmented by emitter-followers at the inputs. This particular topology has moderate noise performance, since the central differential pair is connected *directly* to the emitter-follower outputs without resistances; thus, the total noise of this topology is lower than that of other possible arrangements. The offset voltages are generated across base resistors R_{OS} by currents I_{OS} , the latter also serving to bias the emitter followers. Typical values are $I_{OS} = 1 \text{ mAP}$, $R_{OS1} = 64 \text{ }\Omega$, $R_{OS2} = 72 \text{ }\Omega$, and the three inner tail currents are 75% of the outer tail currents, I_T ($K = 0.75$). The g_m is 10.85 dB below that of the differential pair for the same total tail current, and is flat to within ± 0.15 dB for dc inputs up to ± 137.5 mVP; for a sinewave excitation, the V_{1dB} is at -13.3 dBVP.

We should expect the spurious-free dynamic range—not merely the signal capacity—to improve with the order N , using the following line of reason. Consider again the system shown in Fig. 1, having progressively increasing offsets, with each pair operating at a tail current I_T . For the condition $V_{IN} = 0$, these offsets have the effect of greatly reducing (in the outermost pairs, essentially eliminating) the g_m contribution of all pairs other than the central pair. The contribution of the offset cells to the total noise, due to their intrinsic shot noise mechanisms, is thus diminished: for the outermost pairs, it is almost zero. (This assumes that the noise of the supporting current sources is very low, a condition which can be attained in a well-designed implementation of these principles).

Now, as V_{IN} increases, other pairs become active sequentially, so providing the desired extended linear amplitude response. But each of these pairs has the capacity to contribute significant noise to the output only over a fraction of the total input voltage range. Thus, high-order multi-tanh cells have the potential for achieving a noise level that is always comparable to that of the basic differential pair at the discrete tail current

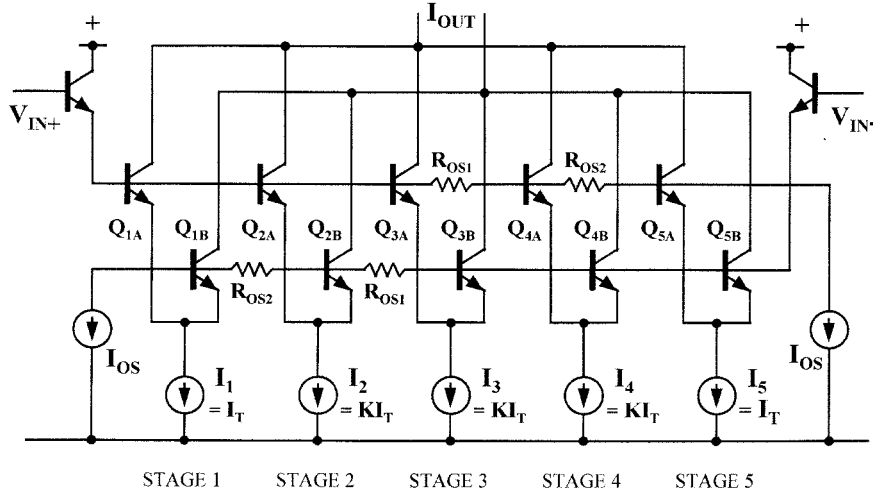


Fig. 2. A method for introducing the offset voltages.

I_T , but where the input-amplitude capacity of the total circuit can, in principle, be increased without limit, just by adding stages.

The g_m of the Fig. 1 system can be altered in two distinctly different ways. In the first method, all the tail currents are varied by the same factor. This results in a simple linear gain variation when the multi-tanh cell is used as part of a $g_m - R$ amplifier (or, of the tuning frequency, when the cell is used as part of a g_m/C filter). Alternatively, the tail biases are fixed, while the offset voltages, which are spaced at equal intervals ψ (PTAT voltages), are varied. It will be apparent that in this case the ratio of the highest to lowest numerical gains will simply be $N:1$, since when $\psi = 0$ the cell collapses to basically a single-differential pair, so it exhibits in aggregate the g_m of N stages all operating at a tail current of I_T , while when ψ is large (say, about $2V_T$), essentially only one stage is operating at this current for any given region of V_{IN} . Thus, for a 25-stage cell, the gain variation range is $20 \log_{10}(25)$, or 28 dB. The noise spectral density will vary by a factor of, at most, $\sqrt{N}$ over the gain range.

This latter technique provides what may be called an “elastic transconductance,” in the sense that the signal capacity can be varied by ψ , “stretching” the g_m outward along the voltage axis to accommodate a large signal using a high value of ψ , or “relaxing” to a smaller signal capacity with a low value. This particular behavior is useful in many automatic gain control (AGC) and other gain-control applications, since it is a basic necessity that the gain be inversely proportional to the signal amplitude in such systems.

Fig. 3 shows the relative numerical gain for the $N = 25$ case as a function of V_{IN} using equal offsets of $\psi = 0, V_T/4, V_T/2, V_T$, and $2V_T$. It is apparent that the area under the $g_m(V_{IN})$ curve is constant; this can be formally proved from (4). The lower panel shows the periodic ripple in the transconductance as a function of V_{IN} , for the case $\psi = 2V_T$. This ripple is essentially sinusoidal,³ with a spatial (voltage-

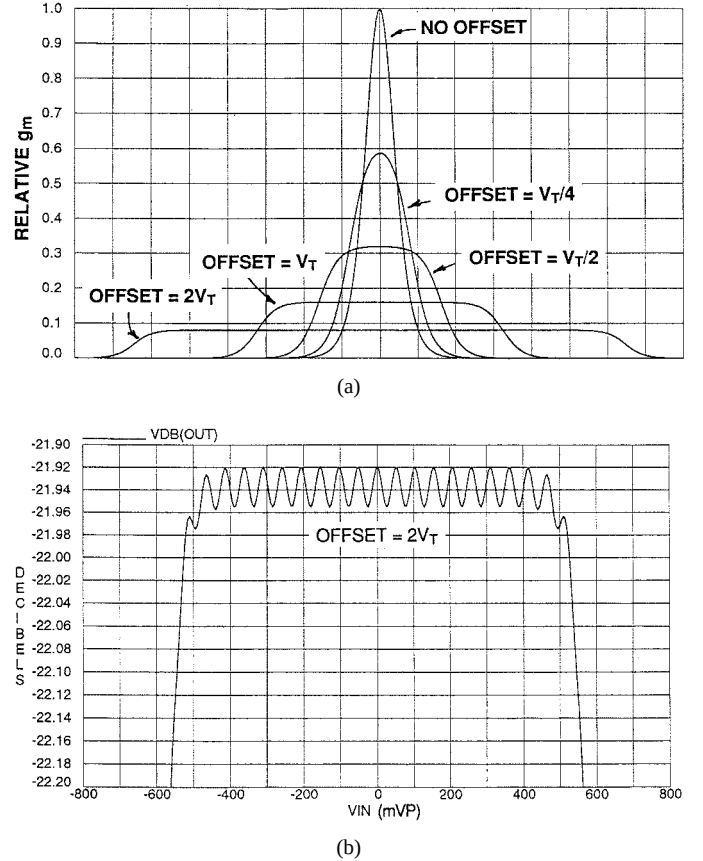


Fig. 3. (a) G_m on a linear scale versus V_{IN} for various offset voltages, $N = 25$. (b) Showing <0.04 dB peak-tp-peak ripple in G_m for $\psi = 2V_T$.

axis) frequency of ψ . Using a result derived from my 1982 paper on trigonometric synthesis using multi-tanh cells [18] the peak-to-peak ripple amplitude can be approximated by

$$20 \log_{10} \{1 + 32 \exp(-1.85\pi^2 V_T/\psi)\} \text{ dB.} \quad (5)$$

The way in which the gain varies with ψ is complex. For small values of ψ , the numerical gain follows $G(\psi) = G_0 \operatorname{sech}\{\psi/V_T\}\sqrt{N}$, while for ψ above about $4V_T$ it is quickly asymptotic to G_0/N , where G_0 is the gain for $\psi = 0$.

³For the ideal case of perfectly matched transistors. Of course, all practical multi-tanh cells will exhibit some random nonlinearities due to emitter-base offsets, or equivalently, mismatches in the offsets introduced using the emitter-follower method. Mismatches in the tail currents will likewise cause spurious nonlinearities, discussed later.

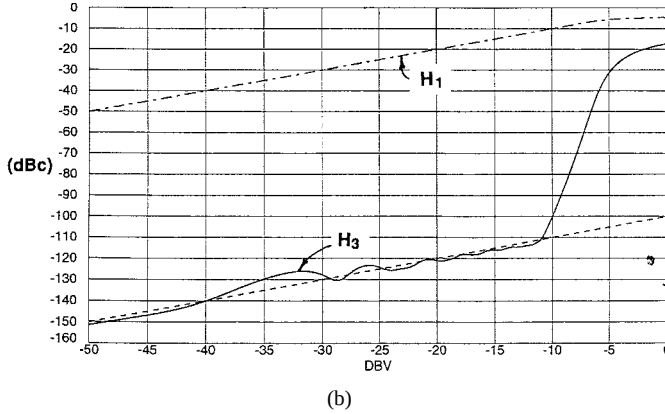
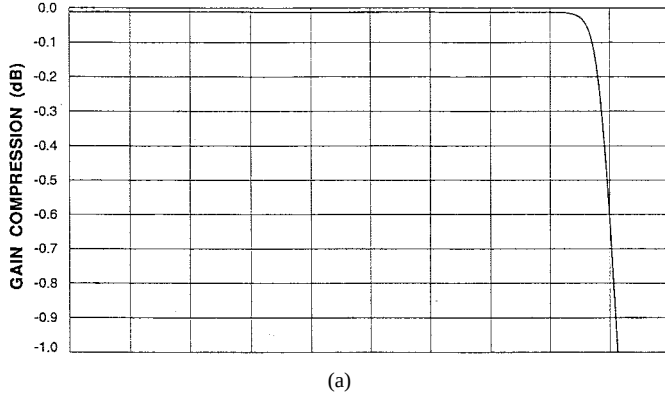


Fig. 4. Harmonic signature of the $N = 25$ multi-tanh: (a) gain compression and (b) showing nonclassical behavior of HD3.

A semi-empirical expression for the decibel gain for large N is

$$G_{dB}(\psi) = G_0 - 20 \log_{10}(N) \{1 - \exp(-\pi\psi/4V_T)\}. \quad (6)$$

The approximation error is within ± 1 dB for all ψ . Clearly, this is not a very attractive gain function, which is one of the several reasons why this type of elastic transconductance is of limited practical value. Van Lieshout and Van de Plassche have described a similar concept [19].

The harmonic signatures for high-order multi-tanh cells are also very complex. The example shown in Fig. 4 is for the circuit of Fig. 1 with $N = 25$ and $\psi = 1.5V_T$. Note that the third harmonic H_3 does not increase in a classical fashion, on a slope of three, but rather on an average slope of one (dotted line) and is a constant 100 dB below the fundamental. A mathematical treatment of this behavior requires the use of Bessel coefficients in the Fourier series expansion and provides limited insight. It is more instructive to employ simulation using idealized transistor models to explore fundamental behavior.

The general case is of considerable academic interest, particularly in the way it forces us to discard classical ideas about distortion based on a simple Taylor-series expansion of the nonlinearity in which the cubic term has a signal-independent coefficient. However, practical multi-tanh embodiments are much simpler. With some additional circuit crafting, we can turn these into very serviceable cells.

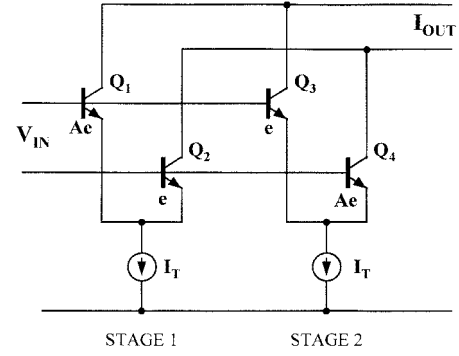


Fig. 5. The basic multi-tanh doubler.

IV. THE MULTI-TANH DOUBLET

Fig. 5 shows the doubler. The individual hyperbolic tangent functions of the paralleled differential pairs $Q1$ – $Q2$ and $Q3$ – $Q4$, which operate at equal tail currents I_T , are offset by making the emitters of $Q1$ and $Q4$ larger than those of $Q2$ and $Q3$. In a monolithic embodiment, the common bases and collectors of $Q1$ – $Q3$ and $Q2$ – $Q4$, respectively, allow the option of realizing the cell in a very compact form using just two pairs of emitters within common collector-base diffusions.⁴

The emitter area ratio A shifts the peak of each g_m by an equivalent offset voltage

$$V_{OS} = V_T \log A. \quad (7)$$

For example, using $A = 4$, the $Q1$ – $Q2$ pair shifts approximately 36 mVP in one direction along the input-voltage axis while the $Q3$ – $Q4$ pair shifts by the same amount in the other. The addition of the two g_m segments, each having the $\text{sech}^2(u)$ form, results in an overall g_m which is much flatter than the simple differential pair, with a resulting improvement in linearity. Fig. 6 shows the individual differential output currents from each pair, their sum, and the individual g_m 's, and their sum, for $A = 4$, $I_T = 1$ mA, and $T = 300$ K. It is apparent that there is some optimum value of A : if too low, the g_m will still have a “hump at the middle”; if too high, the g_m curve will become double-humped. Either condition generates third-harmonic distortion. The maximally flat g_m , that is, the case for which 1) the g_m is never higher than at the point $V_{IN} = 0$ and 2) which theoretically results in zero distortion for a small input, will be shown to occur at a unique value of A .

A. Distortion Analysis of the Doubler

Before proceeding with an enquiry into the optimization of the doubler, it is useful to quantify some basic aspects of

⁴This method of integrated circuit layout, called “superintegration” by the author, was more in evidence in earlier product designs, when simulation and modeling were not so advanced, and when the area consumed by the isolation region surrounding the active base-emitter region of a transistor was much larger than in contemporary processes, particularly since the advent of trench-isolated processes. Under those conditions, the savings in chip area and the reduction in both the total effective C_{JC} and C_{JS} were valuable. Nowadays, superintegration is discouraged, mainly because the benefits are slight and the designer is advised to use model parameters which are generally only available for standard device geometries.

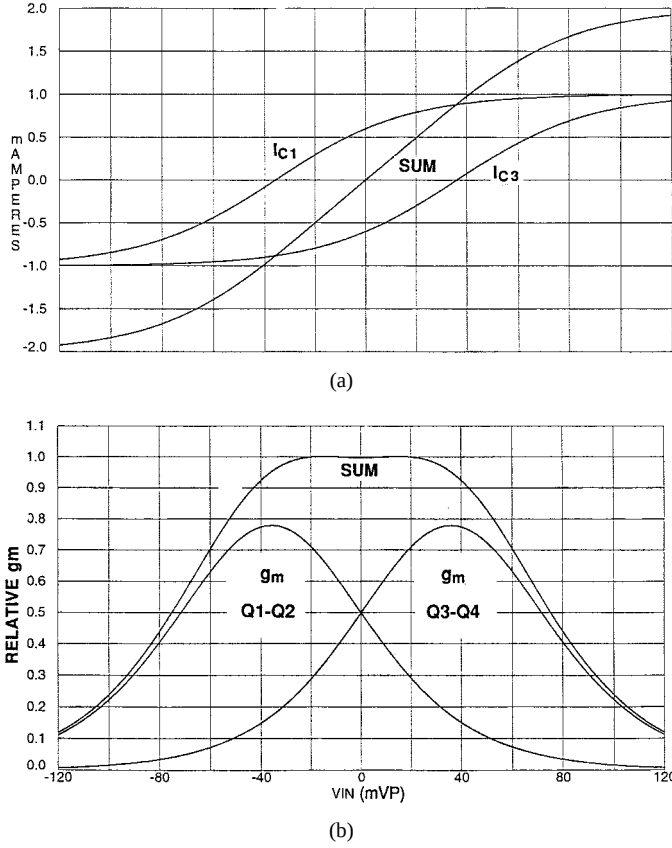


Fig. 6. (a) Collector currents and (b) dual G_m components for the doublet, with $A = 4$.

this interesting circuit. First, we will find an expression for the effective *small-signal* g_m , as a function of the parameter, A . The effective small-signal transconductance (g_m) of one section, say $Q1$ and $Q2$, can be found by considering the incremental r_e 's of each transistor. Note that while the g_m of an *individual transistor* is simply $1/r_e$, the g_m of a differential pair, or an ensemble of such pairs making up a multi-tanh cell, is more complex. We need to keep these distinctions clear. The currents split in the simple ratio

$$I_{C1} = I_T \frac{1}{1+A} \quad \text{and} \quad I_{C2} = I_T \frac{A}{1+A}. \quad (8)$$

Thus we can write

$$r_{e1} = \frac{V_T}{I_T}(1+a) \quad \text{and} \quad r_{e2} = \frac{V_T}{I_T} \frac{(1+A)}{A}.$$

The net g_m is just $1/(r_{e1} + r_{e2})$, which evaluates to

$$g_m = \frac{I_T}{V_T} \frac{A}{(1+A)^2} = g_{mo} \frac{4A}{(1+A)^2} \quad (9)$$

where g_{mo} is the g_m for the simple BJT differential pair. Clearly, the same reduction factor applies to the full doublet. Thus, for $A = 4$, the g_m is reduced by a factor of $16/25$, or 0.64 . We will now find the value of A that results in minimum distortion.

Starting with the general g_m functions given in (1)–(4), and using the simplifying notation $u = V_{IN}/2V_T$ and $\lambda_j =$

$V_j/2V_T$, we have

$$g_m = \frac{1}{2V_T} \sum_{j=1}^N I_j \operatorname{sech}^2(u + \lambda_j). \quad (10)$$

Thus

$$g'_m = \frac{-1}{V_T} \sum_{j=1}^N I_j \operatorname{sech}^2(u + \lambda_j) \tanh(u + \lambda_j) \quad (11)$$

and

$$g''_m = \frac{-1}{V_T} \sum_{j=1}^N I_j \{ \operatorname{sech}^4(u + \lambda_j) - 2 \operatorname{sech}^2(u + \lambda_j) \tanh^2(u + \lambda_j) \}. \quad (12)$$

We wish these derivatives to be zero for zero distortion. The challenge of finding a solution when N is large and for completely general values of λ_j is a considerable one. The problem is greatly simplified by limiting the analysis to low values of N and to the use of symmetric values of the offset factors λ_j and tail currents I_j .

Thus, for the doublet, with only a single offset parameter λ , and at $V_{IN} = u = 0$, (11) becomes

$$g'_m = 0 = \operatorname{sech}^2(\lambda) \tanh(\lambda) + \operatorname{sech}^2(-\lambda) \tanh(-\lambda). \quad (13)$$

Since this is true for all values of λ , due to the symmetry of the tanh function, we move our attention to the second derivative. Setting it to zero

$$g''_m = 0 = \operatorname{sech}^4(\lambda) + \operatorname{sech}^4(-\lambda) - 2 \{ \operatorname{sech}^2(\lambda) \tanh^2(\lambda) + \operatorname{sech}^2(-\lambda) \tanh^2(-\lambda) \} \quad (14)$$

which simplifies to

$$0 = \operatorname{sech}^4(\lambda) - 2 \operatorname{sech}^2(\lambda) \tanh^2(\lambda) \quad (15)$$

the solution to which is

$$\lambda = \sinh^{-1}(1/\sqrt{2}) = 0.658479. \quad (16)$$

The corresponding offset voltage is $2\lambda V_T$, or 34.043 mVP, which can be generated using $A = 3.73205$, from (7). This can be sufficiently approximated using an emitter-area ratio of $3.75 = 15/4$, corresponding to an offset voltage of 34.17 mVP. Employing unit emitters that are the minimum size for the technology, this need not result in excessively large transistors, and in practice, moderately large devices will usually be needed anyway, in order to lower the Johnson noise of the base resistances.

The distortion of this cell is most readily explored using simulation. Fig. 7 shows how the HD3 varies as a function of A for three different amplitudes of sinusoidal excitation. Note that the minima is very deep for small drive levels, and that it shifts slightly upwards at higher drive levels, suggesting that a better practical choice of A may be somewhat higher than that given by the theory. In the examples that follow, we will trace the improvement in the linearity of the fundamental H_1 (and the 1-dB gain-compression level) and in the reduction of

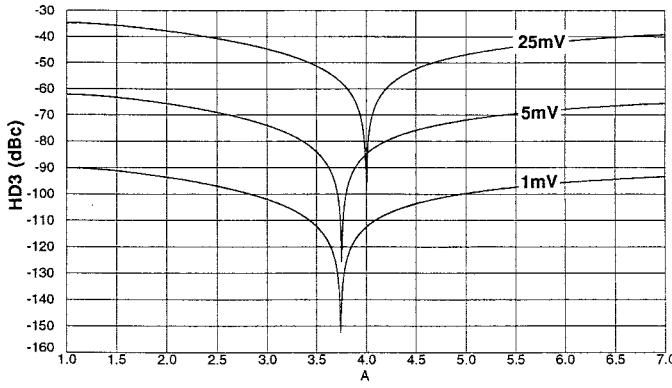


Fig. 7. Variation of HD3 versus A , for sinewave drives of 1, 5, and 25 mV.

the third-harmonic magnitude H_3 using harmonic signatures. Multi-tanh cells are, in principle, free of even-order distortion. However, mismatches of various kinds, notably in the tail currents, will generate even-order terms; these effects will also be investigated.

Fig. 8 shows the harmonic signature for the doublet, using the “practical” minimum-distortion area ratio of 15/4. The third harmonic component H_3 is -110 dB below the fundamental H_1 for an input of -50 dBVP (3.16 mVP sine amplitude), and H_1 experiences a 1-dB gain compression at -22.5 dBVP. This is a considerable improvement over the simple differential pair, for which the corresponding figures are -70 dBc and -28.6 dBVP, respectively. As expected, H_3 does not increase monotonically at a slope of three (dotted line), predicted by simple “cubic” theories of distortion, on which many intermodulation studies are based. Thus, the single-tone third-harmonic intercept IH_3 will be a function of the test level. For the present purposes, we will calculate this intercept extrapolating from a test level of -50 dBVP

$$IH_3 = -50 + (H_{1,-50} - H_{3,-50})/2 \quad (17)$$

where $H_{1,-50}$ and $H_{3,-50}$ are the values of H_1 and H_3 measured at this test level. The IH_3 occurs at $+5$ dBVP. Thus, the two-tone intermodulation intercept theoretically occurs at a level of about 0 dBVP, that is, the two tones would each have a 1-V amplitude at the (extrapolated) intercept.

For the slightly different case $A = 4$, the incremental gain undulates slightly with the instantaneous (dc) input voltage V_{IN} , being up by 0.03 dB at about $V_{IN} = \pm 15$ mVP, and down by -0.03 dB at $V_{IN} = \pm 25$ mVP. (By comparison, the incremental gain of a simple differential pair would be down by -1.957 dB at this dc input level). Using $A = 6$, the 1 dB gain compression is found to occur at -18 dBVP, but the IH_3 is reduced to -12.5 dBVP. These various results provide some indication of how the doublet behaves with regard to distortion, but the most useful insights will be gained by personal experimental simulation studies of one’s own, emphasizing particular operating conditions.

The unit dBV is used for the amplitude of the (sine) excitation, rather than dBm, since these cells are not inherently matched to a source, and are thus not power-responding. However, at high frequencies they may be often matched to a low-impedance source, such as 50Ω , by simple reactive

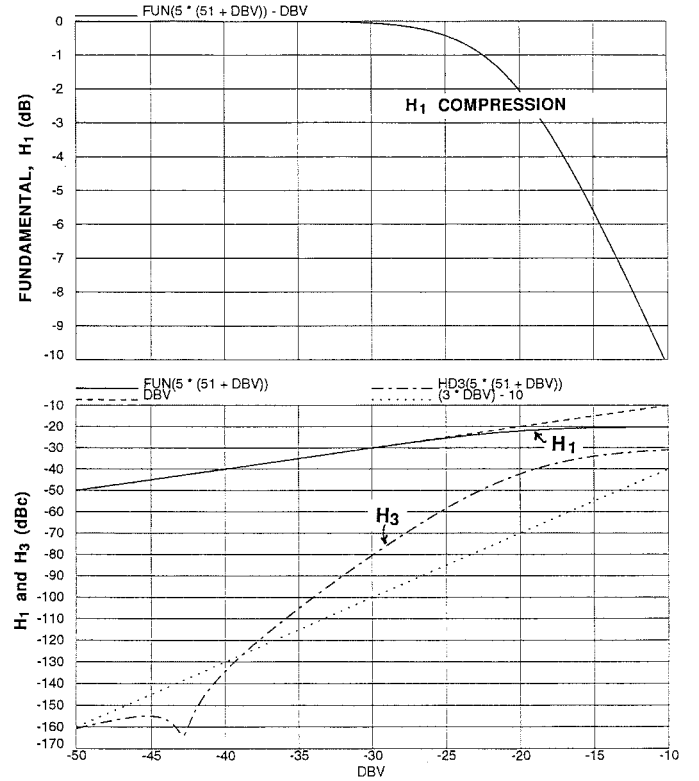


Fig. 8. Harmonic signature of the doublet, with $A = 15/4$.

networks. For the same reason, we consistently state noise-spectral-density in voltage (rather than power) terms. While a more fundamental variable to represent the input is the temperature-normalized ratio $V_{IN}/2V_T$, we will here assume a fixed temperature of 300 K and provide the useful reminder that the input axis scales directly with temperature by using the unit VP (volts-PTAT) for voltages.

Finally, it is important to note that the transconductance of this cell, and thus the numerical gain of an amplifier or mixer based on it, are directly proportional to the tail current I_T , which should be PTAT to maintain a temperature-stable gain.⁵ This allows the development of many types of variable-gain elements; a scheme providing gain control which is “linear in dB” will be described a little later.

B. Noise Analysis of the Doublet

The noise analysis is straightforward. We can begin by assuming that base current noise and the noise due to base resistances are nondominant. The noise voltages appearing in the emitter branches of Q_1 and Q_2 due to the shot noise currents $\sqrt{2qI_C}$ operating on their incremental emitter resistances V_T/I_C are

$$E_{N-Q1} = (V_T/I_{C1})\sqrt{(2qI_{C1})} \quad \text{and} \quad E_{N-Q2} = (V_T/I_{C2})\sqrt{(2qI_{C2})} \quad (18)$$

where I_{C1} and I_{C2} are given by (8). With that substitution, we find from vector-summation that the left-hand section

⁵Following a first-order theory. However, well-designed amplifiers and mixers using these principles will introduce other components to the bias current, to address such practical issues as finite beta and junction resistances.

TABLE I
DYNAMIC RANGE IMPROVEMENTS OF DOUBLET

A	V _{1dB} (dBV)	ΔV _{1dB} (dB)	$\frac{(1+A)}{2\sqrt{A}}$	Δ _{NOISE} (dB)	ΔDR (dB)
1	-28.67	0.00	1.000	0.000	0.00
2	-27.13	1.54	1.061	0.512	1.03
3	-24.50	4.17	1.155	1.252	2.92
4	-21.87	6.80	1.250	1.938	4.86
5	-19.72	8.95	1.342	2.555	6.40
6	-18.04	10.63	1.429	3.100	7.53
8	-15.54	13.13	1.591	4.033	9.10
10	-13.70	14.97	1.739	4.807	10.16

generates

$$E_{LH} = V_T \frac{1+A}{\sqrt{A}} \sqrt{(2q/I_T)}. \quad (19)$$

The noise for the right-hand section is identical. RMS-summing the two noise generators and rewriting in evaluated form, the noise of the doublet is

$$E_{TN} = \frac{0.327 \text{ nV}/\sqrt{\text{Hz}}}{\sqrt{I_T}} \frac{1+A}{\sqrt{A}} \quad (I_T \text{ in mA}, T = 300 \text{ K}), \quad (20)$$

For example, using $A = 1$, corresponding to a simple differential pair, with $I_T = 1 \text{ mA}$, that is a *total* tail current of 2 mA, the noise spectral density is $0.654 \text{ nV}/\sqrt{\text{Hz}}$, while for the doublet with $A = 4$ it is 1.25 times, or 1.94 dB higher, at $0.817 \text{ nV}/\sqrt{\text{Hz}}$. Using this result, and the values for the 1 dB gain compression (V_{1dB}), we can calculate the improvement in dynamic range ΔDR relative to the DR for the basic differential pair. It must be noted, however, that the return of significant amounts of H_3 above $A = 4$ lessens the value of this metric. Table I shows some typical values.

A more complete assessment of noise must include first, the Johnson noise of the base resistances, r'_{bb} , second, the effect of base current noise acting on a finite source impedance, and third, the effect of uncorrelated noise in practical tail-current generators. None of these pose any analysis difficulties, and a detailed discussion is omitted from this overview solely in the interest of brevity and the greater value of presenting a broader variety of interesting topological possibilities.

C. Robustness Issues

There are several factors that need attention in preserving the promise of the doublet in a production environment. IC layout is one of these factors. Common-centroid techniques are routinely used in the layout of g_m cells. Since this involves doubling the number of transistor sites, it is useful to consider whether the doublet might be used instead of a simple differential pair, for example, in op-amps, to minimize input-referred distortion, or to achieve slew-rate enhancement [4] by lowering the g_m for zero-signal conditions, while retaining a high tail current. The need for dual current-sources may be troublesome in some applications, and we should note that there is a noise component caused by the conversion

of *just the uncorrelated* component⁶ of the noise in each of these current-sources to a differential form, through the factor $(A-1)/(A+1)$. This problem can be addressed by the use of highly correlated current sources, that is, ones using a common base drive and large amounts of emitter degeneration.

We will briefly consider three other practical issues: 1) the effect of random emitter-area mismatches, 2) the effect of slightly unequal tail currents, and 3) the effect of ohmic resistance introduced mainly by the indirect access to the emitter-base junctions. Our chief concern here is the distortion behavior, since items 1) and 2) will not significantly impact noise performance, and the noise consequences of ohmic resistances are very easily calculated using standard methods.

It is found that small mismatches in the emitter area ratio are benign, inasmuch as they simply alter the effective ratio A_{EFF} to the *average* of both ratios. Thus, if we choose a nominal value of $A = 3.75$, but experience a worst-case increase of +2% in *just one* of the sets (corresponding to a “ V_{BE} offset” of $\sim 0.5 \text{ mVP}$), the result would be to increase A_{EFF} by +1%. No new distortion components are generated, other than the alteration resulting from the change in A_{EFF} . In the case where both sets vary by some small amount, but in opposing directions, say +2% and -2%, the results remains the same (A_{EFF} is again the average of the two ratios) but the outcome is even more benign, since this average now back to the design value. Very large skews would not be so tolerable, but these will not occur in a well-controlled process.

The effect of slightly unequal tail currents is a little more interesting. A moment’s reflection will show that the addition of the two $\text{sech}^2(u)$ functions results in a linear tilt in the composite g_m near $V_{IN} = 0$. This translates to a parabolic component of nonlinearity. Thus, we can predict some second-harmonic distortion, HD2, but should not expect any change in HD3, since the tilting of the g_m function does not add any parabolic curvature, which would become cubic in the large-signal domain. The magnitude of HD2 can be calculated quite easily. It is small, being -91 dBc for a 1 mV sine excitation, when the currents mismatch by 1%, and (as might be expected, for this particular distortion mechanism) increases by 1 dB for every 1 dB increase in amplitude. These predictions are borne out by simulation experiments.

⁶It is fairly easy to see that the correlated noise components recombine in the output circuit in a canceling fashion.

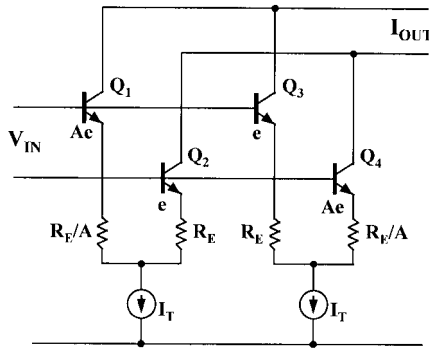


Fig. 9. Multi-tanh doublet with emitter resistances: a hybrid doublet.

This leaves ohmic (junction) resistances. From a noise perspective, the most troublesome will be the $r_{bb'}$ of the smaller transistors. Here, we are more interested in the possible degradation of linearity. Both the base and emitter ohmic resistances will scale with device size, and for the present purpose they can be referred to the emitter branches, as shown in Fig. 9. Rather than attempting a theoretical analysis,⁷ simulations are used to explore this situation, using a simplified set of BJT model parameters. If a more complete set of model parameters is used, the source of the observed effects is unclear, and useful insights are obscured.

D. Hybrid Doublets

We find that small amounts of ohmic resistance have a negligible effect on the linearity. However, this avenue of inquiry leads us to the natural question: Can we achieve another set of optima by treating the exploratory circuit of Fig. 9 as a gift in disguise, and *deliberately* include emitter resistors to improve the linearity? We discover that this is indeed possible. Fig. 10 shows the incremental g_m versus V_{IN} for various values of A and R_E (with a fixed value of $I_T = 500 \mu A$). Most noteworthy is the fact that there is now a three-humped, rippling g_m function, of the sort we will later encounter in connection with the triplet. The peak-to-peak ripple magnitude is 0.007 dB, for $A = 8, R_E = 138 \Omega$, for which condition the input noise spectral density is $1.86 \text{ nV}/\sqrt{\text{Hz}}$. As A is increased (to 8.5 and 9) the gain ripple also increases, through 0.018 and 0.034 dB. It is 0.057 dB for $A = 9.5, R_E = 193 \Omega$, at which point the noise is $2.06 \text{ nV}/\sqrt{\text{Hz}}$. An obvious limitation of this topology is that the g_m can no longer be a linear function of the tail current.

A further valuable alternative, another “hybrid doublet,” uses a single current-source which is split by equal resistors, as shown in Fig. 11. The coupling of the emitters will affect the optimal (minimum-distortion) value of the emitter-area ratio, and we can predict that it will *increase* this value, since when R_E is very small the circuit converges back to a simple differential pair. On the other hand, this topology becomes identical to an emitter-degenerated g_m cell for extreme values of A .

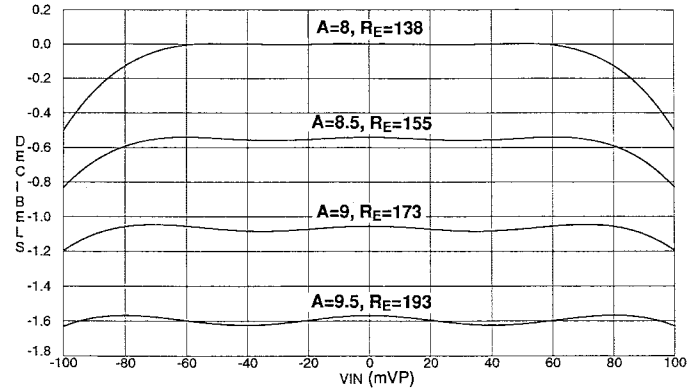


Fig. 10. G_m versus V_{IN} for four cases, using $I_T = 0.5 \text{ mAP}$. Top to bottom: $A = 8, R_E = 138 \Omega$; $A = 8.5, R_E = 155 \Omega$; $A = 9, R_E = 173 \Omega$; $A = 9.5, R_E = 193 \Omega$.

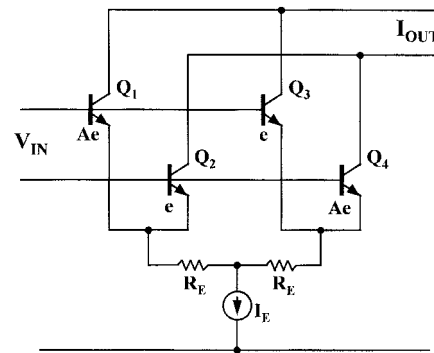


Fig. 11. A variation on the hybrid doublet theme.

The behavior of this cell depends on the zero-signal voltage drop V_E across the resistors. There is again an optimal value for minimum distortion, for which a useful approximation, valid for moderate values of A , is

$$V_E = V_T \left\{ 1 + \frac{4.268}{A - A_0} \right\} \quad (21)$$

where A_0 is the optimal value of 3.732 from (16). For values of A above 50, V_E decreases again. Using a moderate $A = 10, I_T = 2 \text{ mAP}$, and $R_E = 44.5 \Omega$, the V_{1dB} occurs at -20.9 dBVP , while the IH_3 is now at -0.8 dBVP . The noise is 1.46 dB higher than for the doublet with $A = 4$, operating at the same total tail current.

This cell exhibits some amazing properties for large values of A . For example, Fig. 12 shows the small-signal gain, the gain ripple, and noise versus V_{IN} for the case $A = 500, R_E = 63.33 \Omega, I_E = 2 \text{ mAP}$. The gain is extremely flat; for ideal, matched transistors it is within $\pm 0.0015 \text{ dB}$ over the central 200 mVp of input range. The harmonic signature (Fig. 13) shows that the V_{1dB} occurs at -11.2 dBV and the H_3 remains under -120 dBc for inputs up to -40 dBVP , corresponding to an IH_3 of $+20 \text{ dBVP}$. The noise at $V_{IN} = 0$, which now includes that of the emitter resistors, is only $1.83 \text{ nV}/\sqrt{\text{Hz}}$ and is very flat.

Other solutions are given in Table II, in which the noise and dynamic range improvement are relative to the case $A = A_0 = 3.732$; the change in g_m is also noted. In this circuit, any common-mode noise in the current-source will be of little

⁷ As is generally true when translinear circuits are augmented by resistances, the equations quickly become intractable.

TABLE II
PERFORMANCE OF HYBRID DOUBLET

A	V_E (mVP)	V_{1dB} (dBV)	Δ_{NOISE} (dB)	Δ_{DR} (dB)	Δ_{GM} (dB)
4	439.7	-22.38	0.16	-0.12	-0.09
5	112.3	-22.00	0.62	-0.12	-0.37
6	74.06	-21.69	0.94	-0.13	-0.61
8	52.02	-21.21	1.37	-0.08	-0.98
10	44.53	-20.84	1.67	-0.01	-1.26
20	36.90	-19.66	2.49	+0.35	-2.18
50	38.58	-18.03	3.48	+0.99	-3.47
100	43.62	-16.74	4.20	+1.56	-4.51
200	51.02	-13.51	4.92	+4.07	-5.61
500	64.27	-13.52	5.86	+4.06	-7.15
1000	77.13	-12.07	6.59	+3.84	-8.38

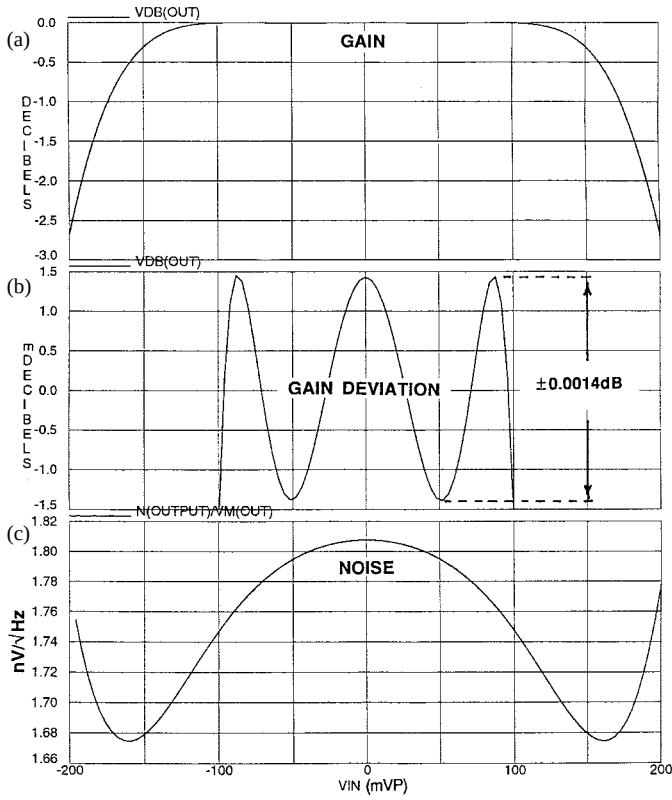


Fig. 12. (a) Incremental gain, (b) expanded gain (showing ± 0.0015 dB ripple), and (c) noise spectral density versus V_{IN} for an optimal hybrid doublet.

concern in most applications. Consequently, this cell not only has far better linearity than the equivalent emitter degenerated pair (obtained by simply removing the large transistors $Q1$ and $Q4$) but also lower noise, particularly in comparison to the simple doublet using dual current sources without emitter degeneration (which have a high level of uncorrelated noise). It is a rare example of a “win-win” situation.

Clearly, a value of $A = 500$ is impractical, if pursued directly, using emitter area scaling. It would result in huge parasitic capacitances and low current densities with a corresponding loss of f_T . The solution is to introduce the equivalent offset directly in the voltage domain, noting that $V_T \log 500$ is a modest 160 mVP. This is most simply implemented using

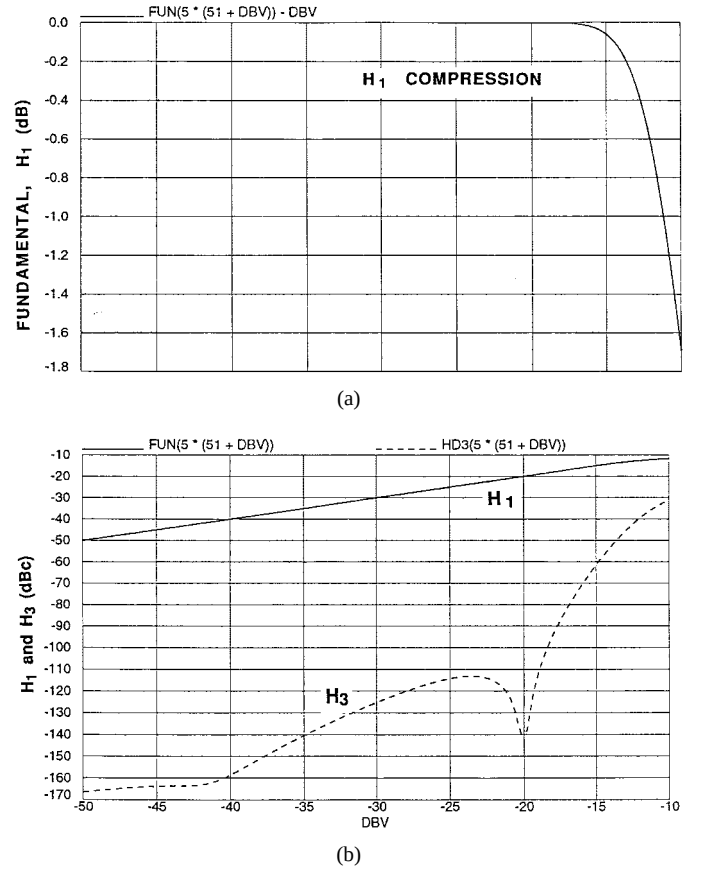


Fig. 13. Optimal hybrid doublet: (a) gain compression and (b) harmonic signature.

emitter-followers, as shown in Fig. 14. The emitter-follower method of creating a multi-tanh cell was proposed by the author and adopted by Gold [5] in 1988, for use in tunable continuous-time filters.

The input noise is somewhat increased, both due to the shot noise in the emitter-followers and the Johnson noise of their base resistances, and that due to the offset-generating resistors R_{OS} . To address this issue, the required effective value of A_{EFF} can be partially provided by a “real” emitter-area ratio A in the multi-tanh section and partially by the voltages $V_{OS} = I_{OS}R_{OS}$, which of course must be PTAT to

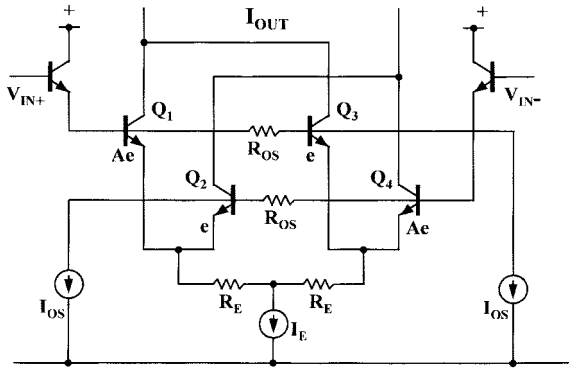


Fig. 14. A practical ultra-low distortion hybrid doublet.

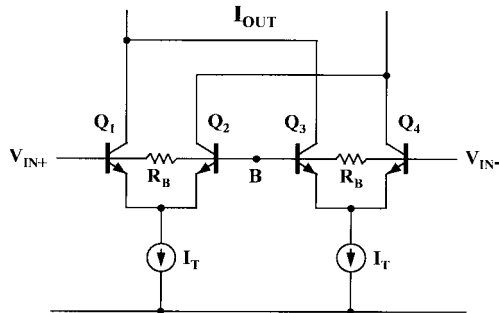


Fig. 15. A basic series-connected doublet.

maintain low distortion over temperature. For example, for $A_{EFF} = 478$, one might use an emitter-area ratio of ten combined with $V_{OS} = 100$ mVP. Since the effective value of A can now be varied through control of I_{OS} , this may also be used to dynamically adjust the shape of the transfer function. The hybrid doublet has successfully been employed in an experimental high-performance UHF mixer.

E. The Series-Connected Doublet

Variants of the multi-tanh concept have been devised in which the offset subcells are connected in series, rather than parallel. Fig. 15 shows the *series-connected doublet*. It will be apparent that this circuit has exactly twice the signal capacity as the parallel doublet, since resistors R_B divide V_{IN} into two equal parts, but it is less obvious that there is no net noise contribution from either the Johnson noise of these resistors or from the base shot-noise currents from $Q2$ and $Q3$ which sum into the center base node B provided that the source impedances at both input nodes are equal. This is because any *common-mode* noise at this node causes equal but opposite-phase noise currents in the inner transistors. A moderate mismatch in the source impedances does not seriously impair this noise cancellation. Consequently, the dynamic range (SNR) performance of this cell is similar to that of the parallel doublet. This is a significant result, since the noise penalty using these same resistors as a voltage divider, driving a parallel doublet, would be much more severe.

There are various other appealing aspects of this cell. One is that it retains the benefit of precise tail-controlled gain. Another is that, if we choose to place the large-emitter devices

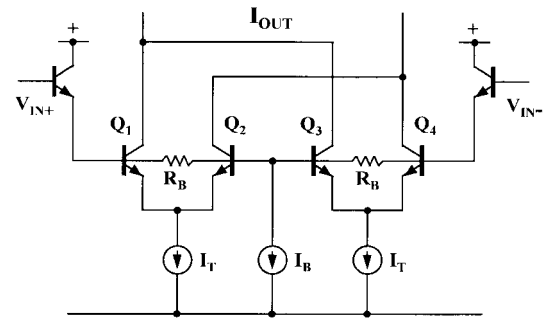


Fig. 16. A practical series-connected doublet.

on the inside, the HF displacement currents in the (large) C_{JC} 's of $Q2$ and $Q3$ cancel, leaving (in the case of a single-sided drive) only the ac current in the (smaller) C_{JC} of $Q1$, having a minimal effect on the HF response. We also have the choice of discarding the outer collector currents (if the halving of g_m is tolerable) and thus eliminating the asymmetric HF signal coupling via the C_{JC} of $Q1$ when driven in a single-sided manner, which generates a troublesome right-plane zero.

We can generate A_{EFF} , the effective value of A , by introducing the equivalent offset voltages, this time, through a single current I_B applied at node " B ," thus generating offsets of $V_B = I_B R_B / 2$. Further, we have the choice of either applying this current *into* the node, in which case the inner transistors appear to have an $A_{EFF} = \exp(V_B / V_T)$, or extract it *from* the node, making A_{EFF} appear in the outer transistors.

We can play on this theme in another way, as shown in Fig. 16. Here, the current I_B also serves to bias the emitter followers $Q5$ and $Q6$, which are included to raise the incremental input resistance, since this was lowered by the inclusion of the base resistors in forming the series-connected multi-tanh cell. The restively loaded emitter followers also introduce a small amount of odd-order distortion, leading to slight gain compression for large inputs. However, this can be first-order compensated by using a larger value of A , which normally would cause slight gain expansion at large inputs. As an illustrative example of this type of doublet, by setting $I_B = 250$ μ AP and $R_B = 360$ Ω ($A_{EFF} = 5.7$) the V_{1dB} is found to occur at -16 dBVP, the H_3 is -125 dBc at a test input of -45 dBVP, and the input-referred noise spectral density is 4 nV/ $\sqrt{\text{Hz}}$. For an ac beta of 100, the incremental input resistance would be about 100 k Ω .

F. Further Forms

The small error in the effective value of A due to the base currents of $Q2$ and $Q3$ in the last example can be eliminated by using two series-connected doublets operating in full parallel, but having opposite polarities of I_B . A yet further simple topology is shown in Fig. 17. This is not strictly a doublet, since it uses only three transistors; however, this interesting cell can be derived experimentally from the circuit of Fig. 15, by first connecting together the two common-emitter nodes and adjusting the area-ratio for minimum distortion. This is in fact the simplest of a family of common-emitter cells; see also [18].

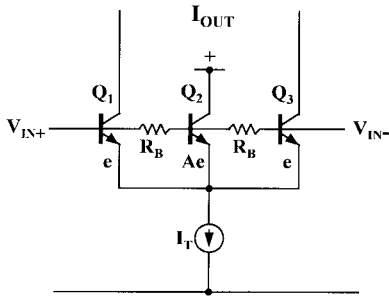


Fig. 17. A three-transistor hybrid form.

It will be apparent that the currents in the two inner transistors are now always equal, and therefore do not contribute to the differential output; accordingly, their outputs can be discarded to the positive supply. As in the series-connected doublet, the base resistances do not contribute to the input noise (provided the source impedances are either low or left-right balanced). On the other hand, as in the hybrid doublet, it uses a single tail current and thus does not suffer from the bias-induced noise of the simple doublet.

Analysis of this cell shows that the minimum-distortion condition now calls for the area ratio A to be exactly four. Using this value, the voltage range of this cell is the same as that for the series-connected doublet; the incremental g_m is down by 2 dB at $V_{IN} = \pm 100$ mVp. Its noise for a total tail current of 1 mA is $1.6 \text{ nV}/\sqrt{\text{Hz}}$, which is not twice that for the doublet ($1.15 \text{ nV}/\sqrt{\text{Hz}}$ with $A = 3.75$). Thus, it has higher dynamic range. Finally, unlike the hybrid doublet, the g_m of this cell remains proportional to the tail current. It is a useful alternative in many applications. Higher order versions of this style of g_m cell have also been devised.

This section has shown that very considerable benefits in linearity can be achieved using cell structures which are not much more complex than a simple differential pair. This improvement comes without a serious elevation of input-referred noise, when low-noise current-sources are employed. Thus, it affords much higher dynamic ranges than simple differential pairs, even when such are aided by emitter degeneration. Also, the valuable property of a transconductance that is proportional to the tail bias current is preserved, in most forms, allowing its use in analog multipliers and variable gain cells. In the next section, further developments are described, including a cell that provides a *linear-in-dB* gain-control interface.

V. THE MULTI-TANH TRIPLET

Fig. 18 shows the simplest realization of the multi-tanh triplet, which has been widely employed in variable-gain mixers and IF stages in dual-conversion receivers for GSM and other communications IC's developed at Analog Devices. It comprises three differential pairs with their inputs and outputs in parallel; the outer pairs have opposing emitter-area ratios of A , larger than for the doublet, and operate at equal tail currents. The inner pair has equal emitter areas (usually not minimum-geometry transistors) so its g_m is centered at $V_{IN} = 0$; the emitter bias current to this center pair is set to K times the outer bias currents, where $K < 1$. As for the doublet, this

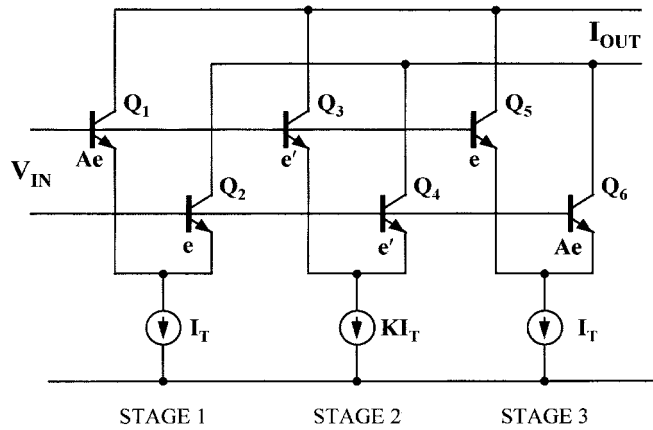


Fig. 18. The multi-tanh triplet.

circuit can optionally be integrated using common base and collector regions, although as noted, this style of layout is generally discouraged in a modern context.

The small-signal incremental g_m for the triplet can be maintained at a nearly constant value over a wider range of input voltage than for the doublet. Its value for $V_{IN} = 0$ can be determined using a simple extension of the theory developed for the doublet case, and is

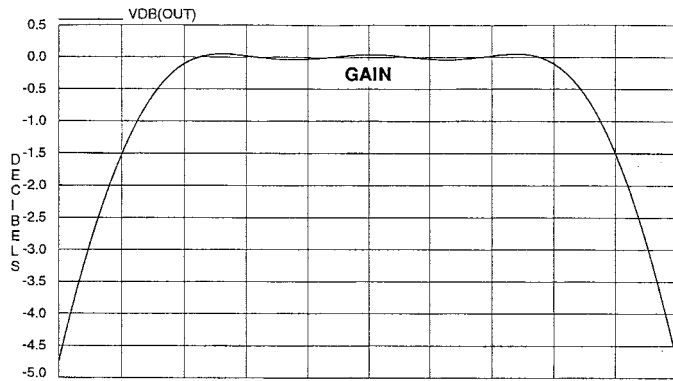
$$g_m = g_{mo} \frac{1}{2 + K} \left\{ \frac{8A}{(1 + A)^2} + K \right\} \quad (22)$$

where, as before, g_{mo} is the g_m that would result with $A = 1$, that is, using the total tail current in a single differential pair. To minimize distortion, we now have to optimize two parameters, A and K . An analytic approach shows an optimum at $A = 7.846$, but the mathematics is complicated and does little to help us visualize the effect on V_{1dB} , IH_3 , and dynamic range. Here, we adopt a pragmatic approach, using simulation to examine all aspects of behavior. In choosing the parameters, we use only integer or low-order rational fractions to ensure robustness in manufacture. These can be chosen in pairs such as to result in an equiripple error in the differential-gain function. Fig. 19 shows this for the case $A = 13$, $K = 3/4$, and is marked to show the definitions used in Table III with various parameters; the noise values are for a *total* tail current of 1 mA. Fig. 20 shows the harmonic signature for this case. It will be apparent that one can again use emitter followers to generate the offset voltages, as in Fig. 2, and construct series-connected versions.

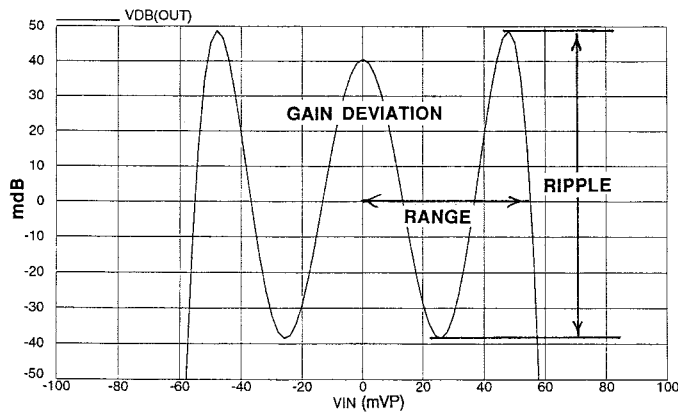
The performance benefits of higher order cells ($N > 3$) are slight for considerable increase in complexity. The *quadlet* ($N = 4$) has two area ratios, A_{INNER} and A_{OUTER} , and an inner/outer current ratio K . Using $A_{INNER} = 4$, $A_{OUTER} = 76$, and $K = 9/11$, the g_m -versus- V_{IN} exhibits an equiripple error of ± 0.2 dB for inputs up to ± 115 mVp; the IH_3 is -14 dBVp and the noise is $1.65 \text{ nV}/\sqrt{\text{Hz}}$ for a total tail current of 1 mA. A version of the *quinlet* ($N = 5$) was shown in Fig. 2; the large outer area ratios ($A = 190$) are more readily implemented using offset voltages. The performance of various series-connected high-order cells have been explored, including developments of the topology shown in Fig. 17.

TABLE III
PERFORMANCE OF SOME OPTIMUM TRIPLETS

A	K	G _{RIFFLE} (dB)	V _{RANGE} (mVP)	V _{NOISE} (nV/ $\sqrt{\text{Hz}}$)	V _{1dB} (dBV)
9	2/3	0.08	32.5	1.286	-19.38
11	7/10	0.06	50.0	1.331	-18.54
13	3/4	0.09	57.5	1.359	-18.11
15	7/9	0.17	65.0	1.384	-17.73
17	4/5	0.26	72.5	1.405	-17.44
19	9/11	0.37	78.0	1.422	-17.22
21	5/6	0.48	84.0	1.434	-17.05



(a)



(b)

Fig. 19. Incremented g_m for multi-tanh triplet, using $A = 13$, $K = 3/4$. Gain, and expanded gain, showing definitions used in Table III.

A. Variable-Configuration Cell

In a moment we will describe a novel combination of the multi-tanh triplet with a special biasing means which has a dual function. Its first function is to convert a linear gain-control current into an exponential one, so as to achieve a linear-in-dB gain law. This general concept may be applied to any of the “fully translinear” multi-tanh cells (that is, those not including resistors in the emitter branches), and is in fact found in several communications products, in mixers, and IF stages.

Its second function is more specific to the triplet case, where a simple modification is introduced that shapes the three bias currents over the gain range. That is, the *relative magnitudes* of the inner to outer currents alter with the absolute bias level.

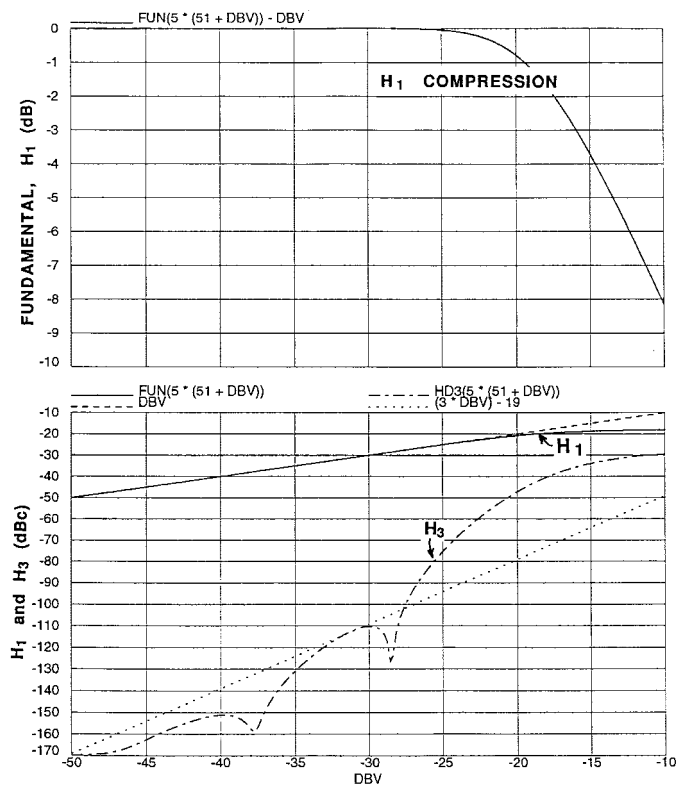


Fig. 20. Harmonic signature for the $A = 13$, $K = 3/4$ triplet.

This causes the triplet to operate as a simple differential pair at high gains (thus exhibiting the lowest noise for a differential structure), while operating in the triplet mode at low gains, exhibiting the extended linear range only when needed.

Fig. 21 shows the basic “linear-in-dB” cell [20]–[22]. The primary current⁸ I_P (PTAT) sets the linear range of the bias current developed at the collector of Q_1 . It is absorbed at the collector of Q_2 , which, together with Q_3 , forms a current mirror. (In a BiCMOS implementation, Q_3 is desirably an NMOS devices, to eliminate the error in I_P due to base current). The emitter area of Q_1 may be raised, so as to linearly scale its current; let its area relative to Q_2 be N . A second

⁸In speaking of currents being PTAT, we are assuming for simplicity that all the resistors used in a complete circuit are temperature-stable, as is the case for thin-film SiCr resistors. While the effect of resistor variations over temperature is to impose a further “shape” on the currents, these effects cancel in determining the overall gain, and the gain-scaling, of a practical circuit, since they invariably occur in ratioed pairs.

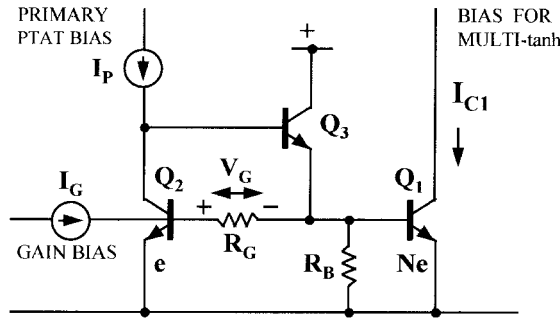


Fig. 21. A basic bias cell generating an exponential current for linear-in-dB gain control applications

PTAT current, I_G , is applied to the base of Q_2 . It flows first in R_G , generating a voltage $V_G = I_G R_G$, and is then absorbed in R_B , incidentally lowering the current in Q_3 to $V_{BE}/R_B - I_G$. It follows that R_B must be chosen to support the maximum I_G at the highest temperature. V_G reduces the current in Q_1 in a simple exponential manner

$$I_{C1} = N I_P \exp(-V_G/V_T). \quad (23)$$

Accordingly, the gain will decrease by 1 dB for each 2.976 mV of V_G since this changes I_{C1} by a factor of $\exp(2.976/25.85)$, or 1.122. Note in passing that a reduction in bias current, hence gain, caused by an increasing I_G is consistent with the general requirements of AGC systems and will result in I_G being directly proportional to a decibel received signal strength indication (RSSI) value.⁹ In practical embodiments of this concept, a further refinement is the inclusion of a simple translinear analog multiplier cell to generate an I_G which is both proportional to temperature and to a temperature-stable gain-control voltage.

Multiple outputs are generated simply by adding transistors sharing the V_{BE} of Q_1 . But it is here that some pitfalls can arise, since the noise currents in these devices comprise both a correlated component—due to everything except the current-sourcing transistors, and in particular, the noise generated across R_G , which is typically 1 k Ω —and an uncorrelated component due to their independent shot-noise and the Johnson noise of their individual $r_{bb'}$. As noted previously in connection with the doublet, these uncorrelated noise components will appear at the cell output multiplied by the factor $(A - 1)/(A + 1)$. The consequences for the triplet are similar, though exacerbated by the higher values of A that are generally used in the outer pairs.

One solution to this problem is to replicate a single output from a bias cell like that in Fig. 21, using highly degenerated current mirrors, which contribute much lower uncorrelated noise. A small practical problem here may be that the available degeneration voltage—at least 10 V_T is desirable—may use up precious supply headroom. This is especially troublesome in an active mixer, when using low (<3 V) supply voltages,

⁹The received signal strength indication voltage is desirably exactly proportional to the decibel power of the signal, accurately scaled (typically 25 mV/dB), and temperature stable. The RSSI function is widely needed in cellular phone systems and other mobile transceivers, where it provides a valuable metric for the control of the transmitted power returned to a base-station and allows this power to be held to the lowest possible value.

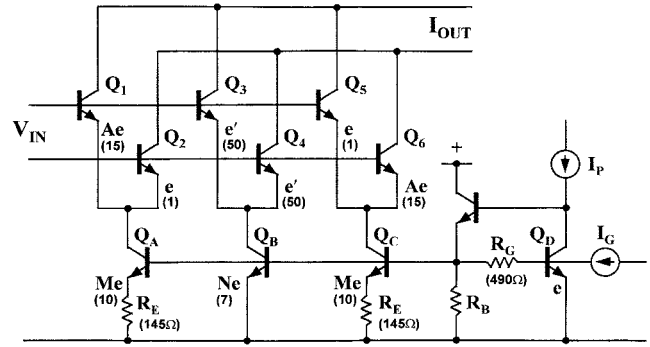


Fig. 22. The “tripplus” G_m cell.

although this method of low-noise tail biasing for doublet and triplet has been successfully employed in mixers and IF amplifier cells in communications IC's that operate at 2.7 V over the full temperature range.

A better solution is afforded by the scheme shown in Fig. 22. Here, a triplet is biased by Q_A , Q_B , and Q_C , whose emitter-area ratios establish the value of K only when the bias currents are at their lowest value, in order to meet the minimum-distortion criteria discussed earlier. This results in an optimum triplet configuration for coping with high-level signals. However, this ratio no longer applies at high bias currents, due to the inclusion of the emitter resistors R_E . In the high-gain condition, the “back-EMF” generated across these resistors greatly diminishes the bias currents in Q_A and Q_C , compared to that in Q_B , and thus in the outer g_m pairs of the triplet. For this condition, the system collapses to essentially a simple differential pair, having minimal noise. For intermediate cases, we have a triplet in which the effective value of K will generally be somewhat too high, leading to different values for noise and distortion than either of the limit cases, but still providing a low-distortion transfer characteristic. This synergistic combination has been called the “tripplus” (a triplet plus optimal biasing). This unique cell implements an “elastic transconductance,” characterized by a constant area under the $g_m - V_{IN}$ curves, in an eminently practical realization.

The optimization space for the tripplus is rather large. We will present some results for a useful case, in which the sizes of the current-source transistors depart slightly from the “ K -criterion.” Accordingly, we use $A = 15$, ten-emitter devices for Q_A and Q_C , a seven-emitter device for Q_B , $R_E = 145 \Omega$, $R_G = 490 \Omega$, and $I_P = 200 \mu\text{A}$. In choosing R_G , we can usefully set it to a value that first-order cancels the effect of base-current losses through the main transconductance cell and typically a mixer core on top of it. We have yet to discover the modified value of I_G needed to cause a 1-dB gain change, or even whether the gain function remains linear-in-dB over some restricted range. It clearly cannot be so over a very wide range, since the control law will become asymptotic to the “ideal” case defined by (23) for very large values of I_G , where the back-EMF in the emitter resistors is too small to affect the bias current ratio.

Fig. 23 shows a set of incremental gain curves for spot values of I_G from zero to 240 μA . The gain varies over a 36-dB

TABLE IV
PERFORMANCE OF THE TRIPLUS AT VARIOUS I_G

I_G (μ AP)	I_{BIAS} (mA)	GAIN (dB)	V_{NOISE} (nV/ $\sqrt{\text{Hz}}$)	V_{1dB} (dB)	IH_3 (dBV)
0	2.054	30.00	0.811	-27.15	-13.63
40	1.143	23.91	1.215	-26.16	-12.90
80	0.649	17.95	1.833	-24.51	-11.85
120	0.366	12.04	2.761	-21.88	-10.43
160	0.201	6.08	4.127	-19.56	-8.67
200	0.106	-0.01	6.102	-18.35	-6.72
240	0.053	-6.28	8.953	-17.73	-4.83

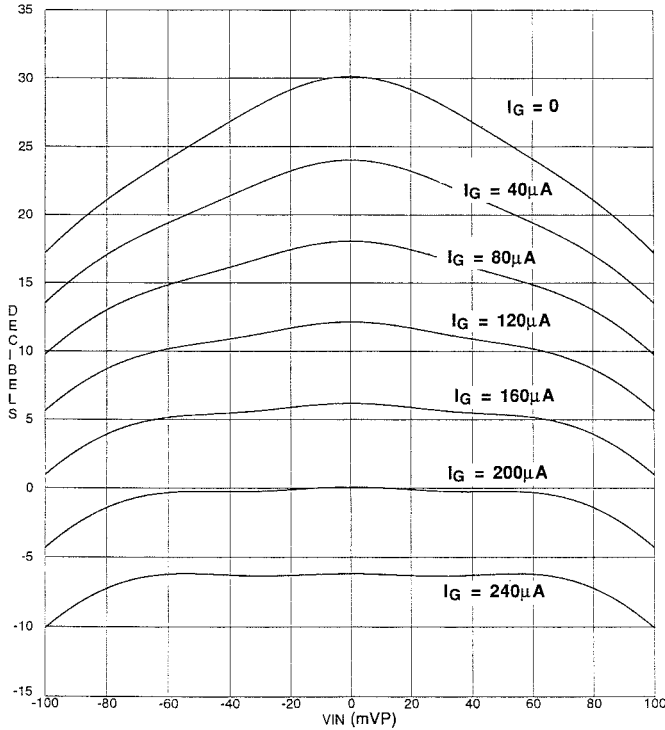


Fig. 23. The incremental G_m versus V_{IN} for an optimal triplus, at various values of the gain-control current I_G .

range, which may be all that is required in some applications. However, it is important to note that in a complete system, say, a high-dynamic range receiver, the triplus might be used not only in a mixer but also in other variable-gain cells in one or more IF amplifier stages. Consequently, the overall gain variation in the complete receiver may be 80 dB or more.¹⁰ It follows that, for the cell under consideration, the linearity requirements at the highest gain would be those for a signal which is some 80 dB smaller than those at the lowest gain; conversely, the factor of 7.5 increase in noise at a gain 30 dB lower will be negligible in the context of a signal some 80 dB (10 000 times) larger.

One further advantage of the triplus deserves mention. In a fixed-configuration triplet, one would need to use transistors of low base resistance throughout, to ensure that the Johnson

¹⁰ See for example, the data-sheets for the Analog Devices AD607, AD5458, AD6459 single-chip receivers, and the AD6432 transceiver, all of which provide this AGC range and are based on the principles discussed in this paper.

noise was low. Unfortunately, this would require that even the smaller transistors in the outer pair would need to be relatively large, resulting in the larger transistors in these pairs having needlessly low $r_{bb'}$, which is of no benefit and introduces excessively high values of C_{JE} , C_{JC} , and C_{JS} , which seriously impair performance at high frequencies. This particular problem might be addressed using emitter-followers to generate the offset voltages, but only with unacceptable noise penalties in a mixer or low-noise IF amplifier application. However, in the variable-configuration cell, the outer differential pairs are almost fully debiased at high gains, and their noise contribution from all sources, including $r_{bb'}$, is negligible. This allows the use of very small transistors in these pairs. The central devices, of course, still need to have low base resistance in critical applications.

Fig. 24 shows the absolute gain and the gain linearity which remains within ± 0.1 dB in spite of the liberties taken with translinear design practice. It further shows the short-circuit input-referred noise versus I_G , which under full gain conditions is reduced to almost the level of a basic differential pair at the same tail current. It is also important to understand that the frequency-dependent base current noise, affecting noise figure in a fully matched mixer, is essentially identical to that of the differential pair, since the outer pairs are strongly debiased at high gains. The modified gain scaling is 0.15 dB/ μ AP, which internally corresponds to a V_G of about 3.27 mV/dB. Table IV summarizes the cell performance at various values of I_G ; the total bias current in the triplet section is I_{BIAS} . The variable-configuration triplet represents a performance high-point in the family of multi-tanh cells.

VI. CONCLUSION

After a long period of relative under-utilization, bipolar multi-tanh g_m cells are now enjoying an increasing number of applications, and have proven practical value in numerous communications products. A few of the novel extensions of the basic concept, and beneficial biasing arrangements, were presented. Many more have been developed, and these cells have numerous other uses in nonlinear function synthesis. As a group, they combine improvements in linearity with wideband operation, in most cases preserving of the useful property of a linear dependence of g_m with bias current, hence, the possibility of precise gain control.

The noise penalties are shown to be moderate, although the figures given here are not completely practical, since they

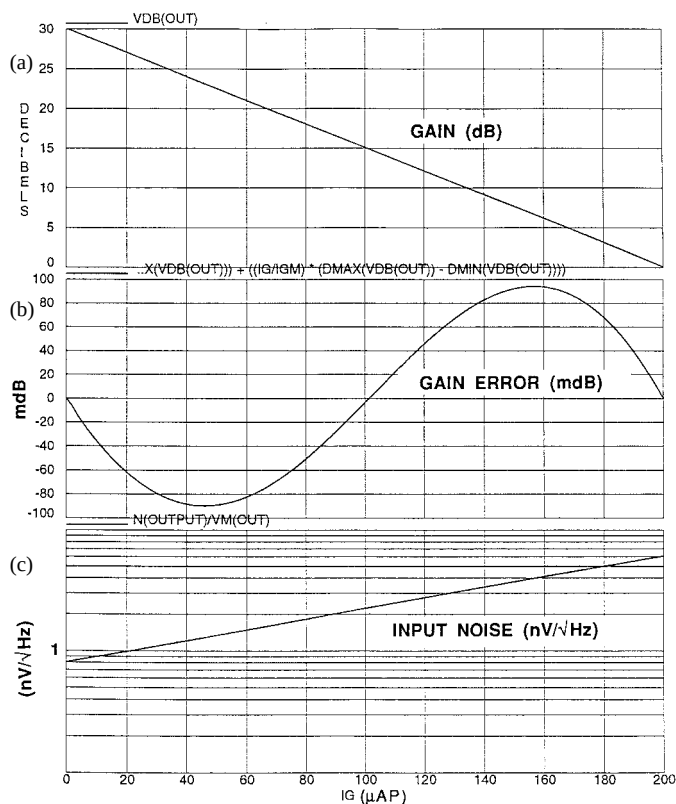


Fig. 24. (a) Absolute gain, (b) gain error and (c) noise spectral density versus I_G for the optimal triplus.

refer only to fundamental shot noise; the Johnson noise of ohmic resistances, in particular, $r_{bb'}$, will often be significant. We have also not fully discussed here the effects of random variations in device sizes (mismatches), which are likely to be more troublesome in elaborate very-high-order cells, presumably chosen because they offer very high linearity. Many aspects of dynamic behavior also need attention in practical designs; these have been omitted in the interests of brevity. These and other issues deserving the close attention of the product designer lie beyond the essentially didactic aims of this overview.

Finally, it will be apparent that these ideas can be translated into MOS form when the devices are used in weak inversion. Since this region of operation extends to useful current levels (microamps) in modern submicrometer devices, there will be practical applications of the multi-tanh principle using pure-CMOS technologies.

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