

A New Technique for Analog Multiplication

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Abstract—A new method is described for performing accurate multiplication of analog signals using devices of special geometry but capable of fabrication with a standard bipolar process. A narrow region of current injection—a carrier domain—can be positioned on an emitter by one electrical input and controlled in magnitude by a second input. The resistive epi layer resolves this current into a differential output proportional to the product of the inputs. A key advantage of these multipliers is their low noise. The basic principle can be applied to many other nonlinear operations.

I. INTRODUCTION

A WIDE VARIETY of methods have been used in the past to perform analog multiplication, many of which involved mixed technologies, for example, magnetic, optical, or electromechanical elements combined with electronics [1]. Currently, the translinear principle [2] is in widespread use, because it is ideally suited to monolithic fabrication, requires no external components, and combines good accuracy and bandwidth with simplicity and low cost. Recent improvements [3] have elevated the accuracy of translinear multipliers to that previously attained only by the best alternative techniques, notably those based on pulse modulation. However, for some applications the noise performance of the basic translinear cell is unsatisfactory [4].

The multiplier described here resulted when a well-known problem in transistor design—emitter current crowding [5]—was turned to advantage. It was the first of several integrated devices in which the function arises directly from the diffusion geometry, rather than by the interconnection of many individual components. The idea is simple: the minority-carrier current injected from a long narrow emitter can be deliberately crowded into a narrow zone, named a carrier domain, by introducing voltage gradients into the base region. By proper design of the base geometry it is possible to induce a voltage which exhibits a maximum potential point whose physical position can be controlled by signals applied to fixed contacts on the base, and thus provide direct control of the domain location in space. This basic concept [6] has obvious applications in A-D conversion, analog waveform generation and function fitting, infinite-hold samplers, and other nonlinear operations. Experimental devices are also being studied in which the domain, acting as a mobile filament of current, moves in response to a magnetic or optical input. Results in these areas will be the subject of future papers.

This paper is limited to a description of the original carrier-domain multiplier [7] which offers a possible alternative to the translinear cell. It can be fabricated with a standard bipolar

IC process, occupies roughly the same chip area, has a response bandwidth extending from dc to about 30 MHz, and requires similar interfacing circuits for conversion from input voltages to differential drive currents and from a differential output current to an output voltage. Its main advantage is that the multiplication process is fundamentally quiet. A second advantage is that the accuracy is not dependent on close matching of transistor V_{BE} and is insensitive to thermal gradients, which by contrast require very careful fabrication and layout of monolithic translinear multipliers. A disadvantage of the design presented here is that relatively large currents are needed to bias the base (2–10 mA) in order to define a narrow domain, and thereby achieve satisfactory linearity. Even then, this design suffers from inherent nonlinearity as a result of departures from the idealized geometry, some of which are discussed in the companion paper by Smith [8], who also describes a layout modification which eliminates some of these sources of nonlinearity, although at the expense of doubling the device area.

Section II provides a description of a basic two-quadrant multiplier and a qualitative explanation of its operation. An analysis of the three main regions is given in Section III using material derived in the Appendixes. The extension to four-quadrant operation follows in Section IV, and a summary of the performance of the four-quadrant design is given in Section V.

II. TWO-QUADRANT MULTIPLIER

Fig. 1(a) shows a plan view of the device, which consists of three main regions.

- 1) A rectangular n-type epitaxial collector, defined by a standard isolation diffusion, about $250\text{ }\mu\text{m}$ by $125\text{ }\mu\text{m}$ in area and of thickness $5\text{ }\mu\text{m}$. A resistivity of $1\text{ }\Omega\cdot\text{cm}$ was used, resulting in a nominal collector sheet resistance of $2\text{ k}\Omega$ per square. Each end of the collector is contacted by n+ diffusions under which bars of buried layer diffusion extend as far as the ends of the emitter.

- 2) A semicircular p-type base diffusion, of junction depth about $2\text{ }\mu\text{m}$ and sheet resistance $200\text{ }\Omega$ per square, contacted at the center and at the two corners.

- 3) An annular n+ emitter diffusion, whose junction depth was controlled to yield an h_{FE} of 40 to 200 and a peak f_T of about 600 MHz. The emitter is contacted by the metallization along its entire length.

The experimental device was fabricated alongside conventional bipolar transistors, and could equally well have used a thick epi process, allowing the concurrent fabrication of the interfacing circuits for operation with standard input and output levels of $\pm 10\text{-V}$ full scale (FS) and $\pm 15\text{-V}$ supplies.

In operation the collector-base and collector-substrate

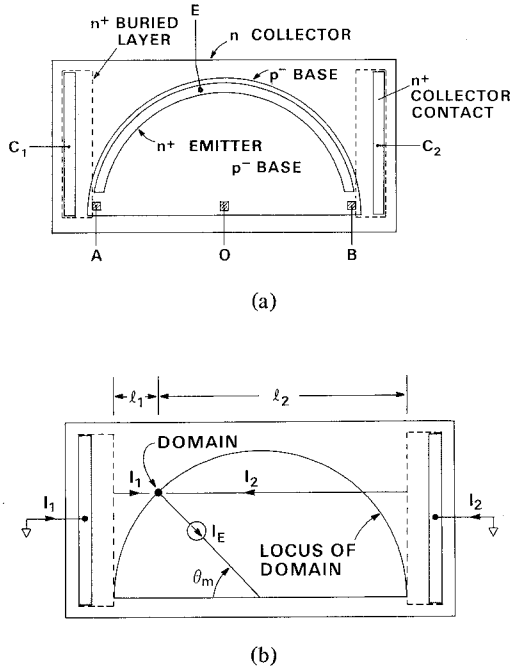


Fig. 1. (a) Layout of the basic two-quadrant multiplier. (b) Geometry of the collector region.

junctions are reverse biased. The epitaxial layer simultaneously serves as the normal sink for injected carriers and as a linear resistive current divider. The base diffusion simultaneously serves as the normal intrinsic base and as a means to generate the voltage gradients required to induce crowding in the emitter, which is driven by a current source I_E proportional to one of the signal inputs. In the absence of bias currents to contacts A and B, the base voltage would be nearly uniform, and injection would occur from all parts of the emitter. Application of negative currents to these contacts establishes a potential distribution in the base which has a pronounced maximum somewhere around the inner edge of the emitter-base junction, which confines emitter injection to that locality and causes most of the emitter to be reverse biased.

A key feature of the base design is its ability to predictably control the domain location. By applying differential currents $-(1-X)I_B$ to contact A and $-(1+X)I_B$ to contact B, where X is a dimensionless variable in the range -1 to $+1$, the angular position of the most positive point around the inner circumference of the base-emitter edge, and hence the angular position of the domain, occurs at

$$\theta_m = \cos^{-1} X \quad (1)$$

where θ_m is measured clockwise from contact A. Contact O supplies the constant current $2I_B$ plus the usual base current due to recombination in the intrinsic (and now mobile) transistor, and it is tied to a fixed voltage point.

Now consider the effect of a point of current entering the resistive collector sheet [Fig. 1(b)]. It splits into two components which subsequently emerge as I_1 and I_2 . Intuitively, we would expect the division of current to be proportional to the perpendicular distances l_1 and l_2 , and this is confirmed by analysis. Thus, the differential output current is

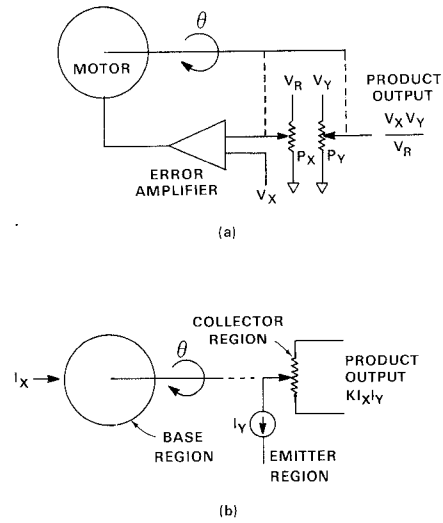


Fig. 2. (a) Classical servomultiplier compared with (b) the corresponding elements of a domain multiplier.

$$I_Z = (I_1 - I_2) = \frac{l_2 - l_1}{l_2 + l_1} I_E \quad (2)$$

where I_E is the current supplied to the emitter terminal. A consideration of the geometry shows that l_1 and l_2 may be replaced by expressions using θ_m which transforms (2) to

$$I_Z = I_E \cos \theta_m. \quad (3)$$

Combining this with the injection angle given by (1) yields

$$I_Z = X I_E \quad (4)$$

that is, the differential output is the linear product of the two inputs, and since X can have either sign two-quadrant operation is possible.

It is useful to compare this device to the classical servomultiplier, to which it bears some interesting similarities. Fig. 2(a) shows a typical electromechanical multiplier, the operation of which is well known [1]. One of the inputs is first converted to a shaft angle, and consequently has a long response time. The output potentiometer P_Y provides almost noise-free voltage division of the input V_Y . In the carrier-domain multiplier, the angular response is about eight orders of magnitude faster and the output potentiometer (formed by the collector sheet, over which the domain moves in analogy to the wiper of P_Y) provides low-noise current division of the input I_E [Fig. 2(b)].

III. ANALYSIS

The first carrier-domain multipliers had decidedly heuristic beginnings. The unique properties of a semicircular resistive dee were discovered during experiments using large models cut from "Teledeltos" resistance paper to which voltage or current sources were attached and the resulting potential distributions

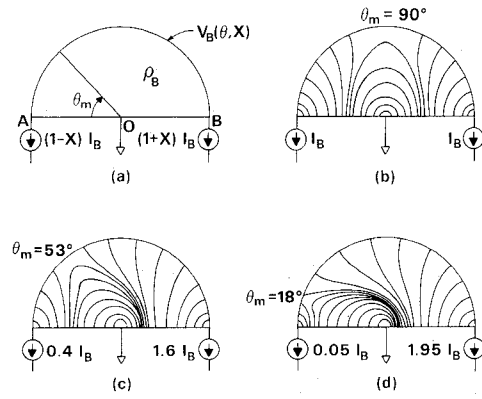


Fig. 3. (a) Idealized base region and approximate equipotential distributions for (b) balanced base drive, $X = 0$, (c) $X = 0.6$, and (d) $X = 0.95$.

measured with a digital voltmeter (DVM). The moving potential maximum at the circumference was first used as a means to steer a current into one of several transistors, as part of a monolithic character generator [9]. It then became apparent that the inverse-cosine relationship between θ_m and X could be combined with the cosine current-division relationship in the collector to form a linear multiplier. The detailed analysis followed much later. The base potential is the key to device behavior so it is discussed first. The emitter current density profile can then be calculated. Finally, the collector region is examined more closely.

Base Potential Distribution

Fig. 3(a) shows an idealized base dee, having point contacts. The conformal transformation given in Appendix I shows that the circumferential potential at a given value of the angular control input X is

$$V_B(\theta, X) = \frac{2\rho_B I_B}{\pi} \left[(1-X) \ln \sin \frac{\theta}{2} + (1+X) \ln \cos \frac{\theta}{2} \right] + C \quad (5)$$

where ρ_B is the base sheet resistance. The inverse-cosine relationship stated in (1) follows from equating the derivative of this expression to zero. True point contacts would result in infinite potentials at the contacts. In practice, the contact areas dictated by processing tolerances are sufficient to keep the contact voltages within sensible bounds, but it was later realized that larger contacts should have been used to avoid unnecessarily large drive voltages, and also to avoid mobility modulation due to the high current densities. The optimization of base contacts is discussed by Smith [8]. The constant C in (5) is determined by the contact radius at 0. Its value need not be known exactly, because the emitter current density profile is independent of the absolute base potential.

Fig. 3(b)–(d) show typical equipotential lines for three values of X . The potentials along the radius at 90° do not depend on X , due to the use of differential drive currents, so the set of curves for $V_B(\theta, X)$ intersect at $\theta_m = 90^\circ$, and the potential at the maximum V_m must be a function of X . Its exact value is derived in Appendix I. In practice, it is con-

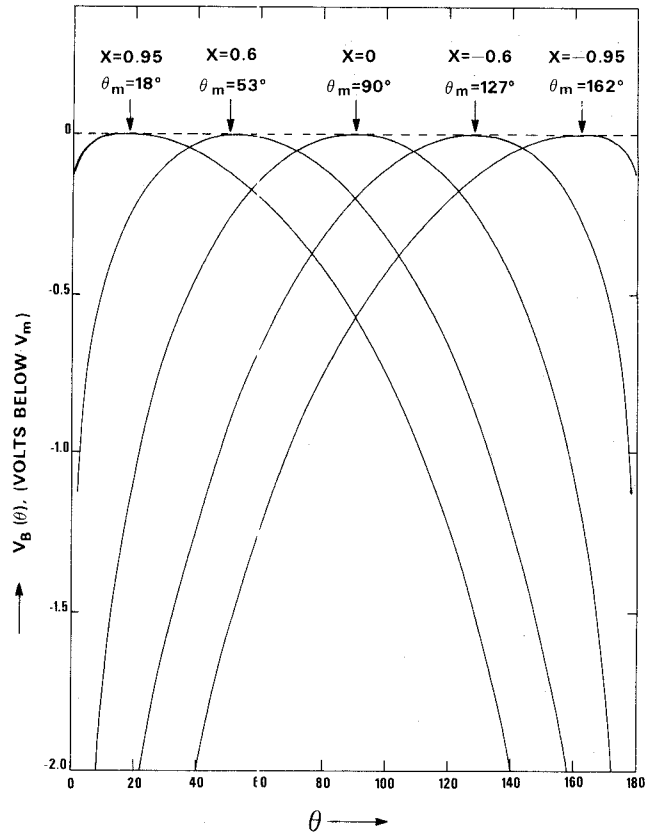


Fig. 4. Potential distribution at circumference of base region for various values of X .

venient to refer the base potential distribution to this peak value and use the adjusted variable

$$V'_B(\theta) = V_B(\theta, X) - V_m. \quad (6)$$

The motion of θ_m from 0° to 180° as X is varied from -1 to $+1$ is clearly seen in the curves of Fig. 4, calculated for $\rho_B = 200 \Omega$ per square and $I_B = 8$ mA.

Emitter Current Density

It is obvious from Fig. 4 that very severe crowding of emitter injection will occur near θ_m . Appendix II derives the expressions for current density in a straight emitter. For an annular emitter

$$J_E(\theta) = J_o \exp \frac{V'_B(\theta)}{V_o} \quad (7)$$

where

$$J_o = \frac{I_E}{R_E W_E \int_0^\pi \exp V'_B(\theta)/V_o d\theta} \quad (8)$$

and R_E is the emitter radius. Using the adjusted values for base potential J_o is also the peak current density. By substituting the expression for the actual base potential (5) a closed-form equation for $J_E(\theta)$ can be derived. However, it is complicated and not very helpful. It is more convenient to take

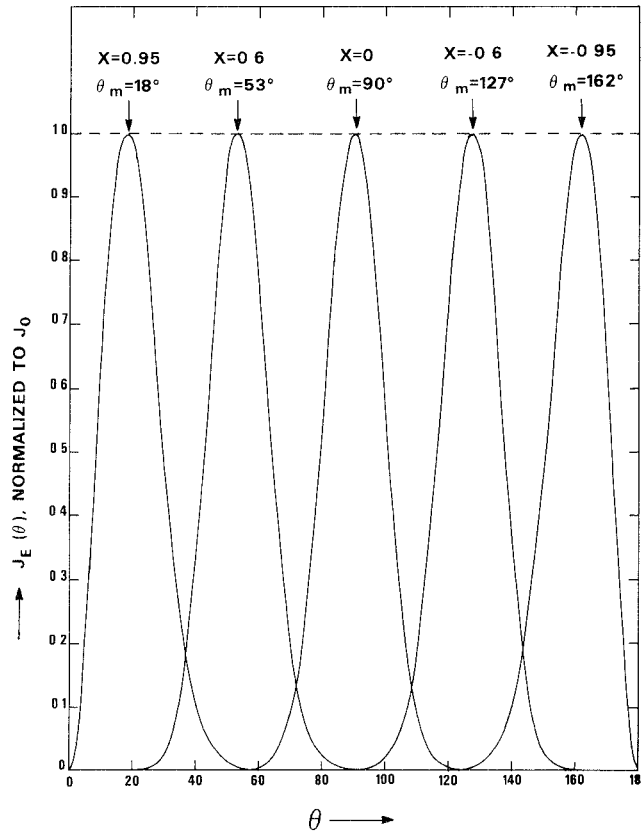


Fig. 5. Emitter current density distributions for bias conditions shown in Fig. 4 and $T_A = 300$ K.

the numerical values for $V'_B(\theta)$ and calculate $J_E(\theta)/J_0$ using (7). The absolute value of J_0 can be found independently (if required) from the approximation given in Appendix III, which also demonstrates that in the region near θ_m the potential distribution is nearly parabolic, giving the domain a Gaussian form.

Fig. 5 shows curves for $J_E(\theta)/J_0$ using the values for $V'_B(\theta)$ plotted in Fig. 4, which actually correspond to a high base drive current, and the domain would typically be more diffuse. Nevertheless, the predicted Gaussian form is visible, and the domain is seen to retain its symmetry over most of the control range. The width of the domain at the half-magnitude points is about $35 \mu\text{m}$ for $R_E = 100 \mu\text{m}$, or 11 percent of the annular length.

The emitter junction will be reverse biased at points remote from θ_m , the greatest stress occurring when $X = \pm 1$ and all of I_B flows in either contact A or B. The drive current should not be allowed to cause emitter breakdown, since this will result in gross nonlinearities.

This simplified analysis assumed no transverse potential gradients in the base under the emitter. In fact, there is a strong transverse gradient. With reference to Fig. 1(a) note that the base can be partitioned into the dee below the emitter and the roughly annular part above it. The latter can be approximated by a simple resistive stripe separated from the dee by the sub-emitter region. If there were no conduction under the emitter, the potential around the annular part of the base would vary linearly from A to B, and a moment's considera-

tion shows that this profile is always more negative than on the lower edge of the base-emitter junction. Consequently, most of the injection occurs along the lower edge, resulting in much higher current densities than predicted by simple theory. Furthermore, injection will tend to occur near the surface (where the base potential is highest). These mechanisms lower the effective alpha of the intrinsic transistor and make it sensitive to surface defects, which may not be uniform over the emitter length. The resulting modulation of alpha as the domain moves around the emitter may be a cause of observed variations in the nonlinearity of individual samples.

The intrinsic transistor operates in essentially a grounded-base configuration, because even though the base voltage is not fixed the emitter is driven from a high-impedance current source. This has several advantages. First, lot-to-lot variations in " V_{BE} " (which here refers to the base-emitter voltage measured with respect to V_m at a total current I_E) will have negligible effect on performance. Second, because the domain occupies very little space at a given time, this V_{BE} can vary considerably from one end of the emitter to the other without causing serious skewing, and linearity is therefore not a strong function of V_{BE} uniformity. Third, large thermal gradients along the emitter can be tolerated, by the same reasoning. Fourth, in the four-quadrant multiplier described later, the two emitters can likewise have a large V_{BE} mismatch and operate at different temperatures with little effect on accuracy. Contrast this with the extreme care required in the design of an accurate translinear cell [3].

The grounded-base operation also results in relatively low-noise operation since the thermal noise voltage of the base resistance does not contribute to the output noise for frequencies up to which the emitter current-source impedance remains high. Here also, the contrast with a translinear cell is noteworthy, for which circuit this noise voltage is multiplied by a factor $\sqrt{2}$ (due to use of the differential pair of transistors) and then translated to an output noise current by the high transconductance of the stage. In this connection, a diffuse domain behaves somewhere between an ideal point domain and a differential pair. Viewed another way, the centroid of the domain suffers a statistical fluctuation due to the thermal voltages in the base. This mechanism has not been analyzed, but it is likely that the noise contribution is reduced in proportion to the projected width of the domain to the perpendicular line between collector contacts.

Collector Current Division

In Section I some assumptions were made about the collector region. They were: 1) the outputs were connected to zero-impedance nodes; 2) the output currents divided in proportion to the distances l_1 and l_2 ; and 3) the domain behaved like a point source of current. These assumptions need closer scrutiny.

The output currents can be directly converted to voltages using load resistors, and the maximum load power is delivered under the usual matching conditions of equal source and load resistance. However, low-impedance current-summing nodes are preferable, because the current gain is maximized, and the supply-voltage requirements and collector circuit time constants are minimized. More important, the scaling accuracy is then independent of the absolute resistance of the collector layer, which may vary by as much as +1 percent/ $^{\circ}\text{C}$ for a $5\ \Omega \cdot \text{cm}$ epi. The loading conditions can be met using an op amp as shown in Fig. 7, or by the use of a differential feedback circuit, such as the one used in [3].

The proportionality between the distances l_1 and l_2 and the output currents can be proved using another conformal transformation. Let the collector rectangle be placed in the z -plane with one long edge lying along the x -axis and one collector contact lying along the y -axis. Using $w = e^z$ this maps to the w -plane as an annulus, and the contacts become a pair of coaxial rings. The distances l_1 and l_2 become radial lengths and vertical position becomes angular position. Hence, the division of the input current is independent of vertical position in the z -plane, and by shrinking this dimension the collector layer reduces to a one-dimensional resistor, for which the proportionality is obvious.

This argument assumes that the collector sheet is uniform. In fact, the epi layer is subject to at least three distortions. First, the layer is thinner under the base than elsewhere. Second, the injected current will cause modulation of the majority-carrier mobility, and therefore of the local sheet resistance, particularly using a lightly-doped epi. Third, the voltages induced in the collector will modulate the depletion-layer width, again varying the local sheet resistance. These nonuniformities are not easily quantified, but of course their net effect is to introduce nonlinearity, both with respect to

the angular signal and, to a lesser extent, to the emitter-current signal. The problem can be reduced by using a special light buried layer diffusion for the entire collector sheet; gridiron and peppered diffusions using a regular buried-layer concentration have been considered. More recent multiplier designs use a long straight emitter, and the domain slides over a narrow bar of regular buried layer forming a "resistance track" of about $1\ \text{k}\Omega$ end-to-end.

To deal with the fact that the domain is not a point of current, but really rather fat, we must show that there is an effective center of injection, whose angular position closely tracks θ_m . This is possible because the diffuse domain can be decomposed into elemental point currents, the total effect of which is found by their superposition in the resistive collector layer. This centroid can be defined in three ways: 1) the point in the emitter plane at which a point source of current, equal in magnitude to the total emitter current I_E , could be substituted without changing the outputs; 2) the intersection of the lines dividing the emitter plane on either side of which exactly half of I_E is injected, which provides the formal definition, 3) in terms of the solution to the equation

$$\int_0^{\theta_c} \int_0^{w_c} J_E(\theta, w) d\theta dw = \int_{\theta_c}^{\pi} \int_{w_c}^{w_E} J_E(\theta, w) d\theta dw \quad (9)$$

where θ_c and w_c are the angular and radial coordinates of the centroid. If the transverse gradient is ignored, this equation can be used to derive an expression for the angular error $\theta_m - \theta_c$. However, a simpler approach can be used to demonstrate that this error is not a serious source of nonlinearity. First, note that the centroid occurs at exactly θ_m as long as the domain is symmetrical about this angle. Fig. 5 shows that this condition is reasonable for a line emitter, and it has already been stated that most of the injection does occur along the inner emitter edge. At $X = \pm 0.95$ there is some skewing, but even here it is mostly in the tails of the current-density distribution. Furthermore, as θ_c approaches 0° or 180° the sensitivity of output current to injection angle approaches zero, since ultimately the domain is moving parallel to the contacts. The central 80 percent of the X -input range translates into an angular range of only 60 percent (37° to 143°). This provides a good explanation of the observed fact that the X -linearity holds up quite well even for drive currents (I_B) down to about 2 mA, one fourth the value used to plot Figs. 4 and 5.

Finally, it should be pointed out that isothermal variations of temperature do not affect the position of the centroid as long as the symmetry condition holds. Probably the largest source of temperature dependence of scaling factor is the variation of α , amounting to about $+0.01$ percent/ $^{\circ}\text{C}$, and this can easily be compensated.

IV. FOUR-QUADRANT MULTIPLIER

Analog multipliers are usually expected to operate in all four quadrants, and this is achieved simply by putting two of the devices just described into a common collector region. Fig. 6 is a photomicrograph of both types. The four-quadrant multiplier has its base regions cross connected and driven from

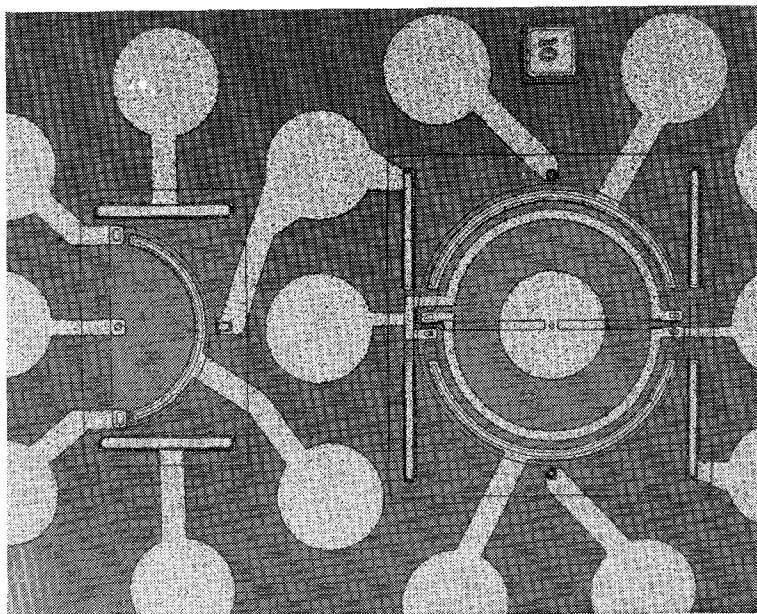


Fig. 6. Photomicrograph of the two-quadrant (left) and four-quadrant (right) carrier-domain multipliers.

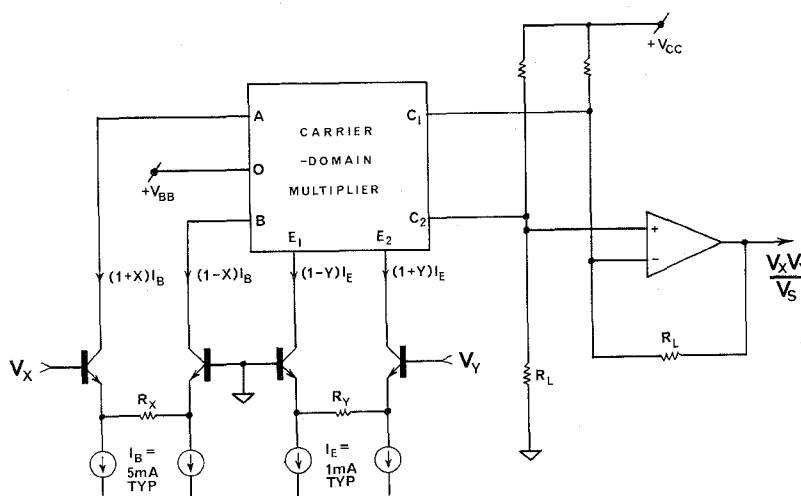


Fig. 7. Typical interface circuitry.

a pair of differential currents modulated by the X input. The common base connection is made to the circular bonding pad at the center; this was convenient, but some of the performance variations between samples may be traceable to the strain induced in the base diffusion by wire bonding to this pad. The two emitters are driven from a pair of differential currents modulated by the Y input. Fig. 7 shows a suitable interfacing circuit.

The operation is as follows. When the emitter currents are balanced ($Y = 0$) the two domains move on arcs at a constant 180° separation in response to the X input, but since they contribute equally to the outputs, the differential output is zero. When the base currents are balanced ($X = 0$) both domains inject into the center of the collector, and again the differential output is zero. The upper half-multiplier generates

the output

$$I_{ZU} = X(1 + Y)I_E \quad (10)$$

and the lower half-multiplier generates

$$I_{ZL} = -X(1 - Y)I_E \quad (11)$$

the negative sign being due to the opposing motion of the domain. The total differential output is therefore

$$I_Z = 2XYI_E \quad \text{for } -1 < X, Y < +1. \quad (12)$$

Using the bridge-feedback output converter shown in Fig. 7, the voltage output can be expressed simply as

$$V_Z = \frac{V_X V_Y}{V_S} \quad (13)$$

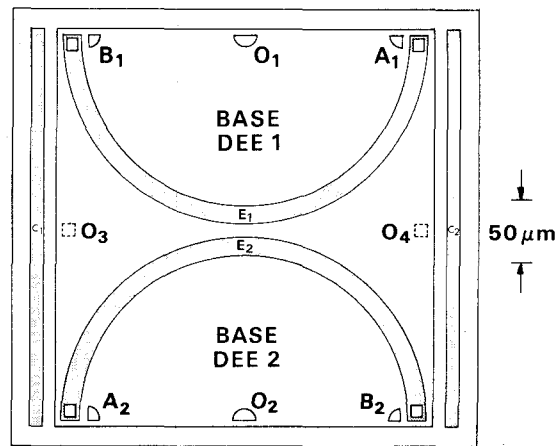


Fig. 8. A modified geometry for the four-quadrant multiplier.

where V_S is the scaling voltage for the complete multiplier, given by

$$V_S = \frac{R_X R_Y I_B}{2R_L} \quad (14)$$

Feedthrough

In any practical four-quadrant multiplier, a small output is generated even when one input is nulled. This "feedthrough" signal is usually specified as the percentage of FS output which appears when one input is driven to $\pm$ FS and the other is adjusted to minimize this output. The carrier-domain multiplier exhibits very low feedthrough on the Y input, because of the good linearity of this channel. A small initial offset in the domain positions can be adjusted by a corresponding offset to the X input until near-perfect cancellation occurs in the linear collector resistance. X feedthrough is much worse, because now cancellation depends on exact matching of the nonlinearities of the two half-multipliers. Local defects can affect each half separately, especially when the domains are at 90° , and hence near the edge of the collector boundary, and this also is the position where the output is most sensitive to an angular error.

Fig. 8 shows a proposed rearrangement of the design, expected to have superior X -feedthrough characteristics, and generally lower distortion. The most conspicuous difference is that the emitter arcs have been reversed. Consequently, when the domains are at their point of maximum sensitivity ($\theta_m = 90^\circ$) they move on very nearly parallel paths in close proximity, and nonuniformities in the epi layer, caused either by processing gradients or by some of the modulation effects described earlier, are much less damaging to linearity. The base region is now rectangular, so the collector is nearly uniform in thickness, and the semicircular base boundaries are now defined by the emitter arcs, using the high sub-emitter sheet resistance as a sort of isolation wall. Transverse crowding is not avoided, but preliminary studies show it may be possible to bias the contacts O_3 and O_4 so as to set up an image profile on the outer edges of the emitters. Note that there is

now no need to put a bonding pad on the base diffusion. Finally, the two emitters are contacted just at their ends. This allows the use of a thinner emitter, thus reducing its transition capacitance. Provided that the emitter currents are no more than a milliamp or two, the voltage drop in the emitter series resistance can be tolerated. It will cause a broadening of the domain, but if both contacts are used, this will be symmetrical.

Transient Response

The step response of the domain multipliers described here is complicated, and can only be described in qualitative terms. The simplest situation occurs when the domains are parked in their limit positions, and so inject over the collector contacts. The elemental transistors then behave like conventional grounded-base stages with almost zero load resistance in the collector circuit. However, there is a large base resistance, formed by the dee, and a fairly large emitter transition capacitance due to the inactive portion of the emitter. These conspire to form a resonant circuit, and therefore a response overshoot or ringing.

When the X input is balanced the domains are stationary over the center of the collector. In addition to the effects just described, there is now a delay and rise time contribution due to the distributed RC network formed by the epi sheet resistance and the base- and substrate-junction capacitances. It may be concluded that the response on the Y input will be fast for $X = \pm 1$, and slowest for $X = 0$.

The response on the X input is more complex. Here, the input step must first change the potential distribution on the base dees, which are again sluggish by distributed RC effects, although less seriously due to the order of magnitude lower base sheet resistance. Some of the resulting displacement currents appear quickly at the collector terminals, due to imperfect cancellation, and cause preshoot. Next, the domains have to follow the moving potential peak, and finally, their currents must traverse the collector network before appearing as output. No attempt has been made to analyze these mechanisms. However, in most practical cases the overall response time will be dominated by the interface circuitry, because as the mea-

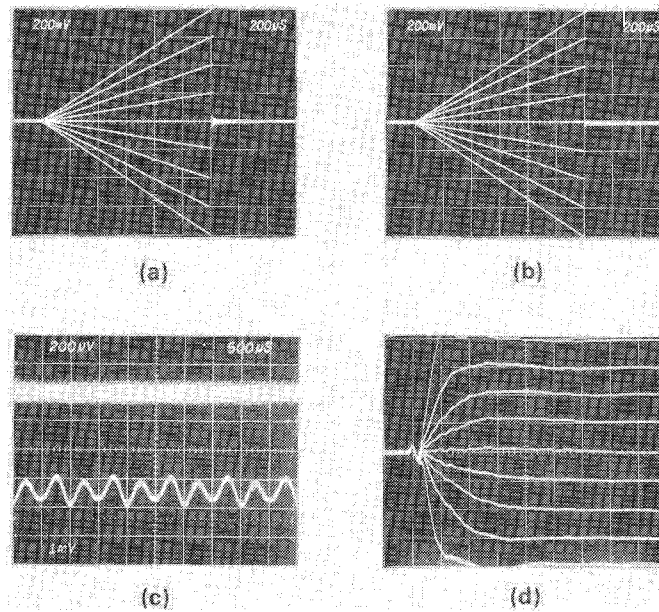


Fig. 9. Typical performance, showing output for (a) ramps applied to bases. (b) Ramps to emitters. (c) Feedthrough for Y (upper trace) and X (lower trace) inputs. (d) Transient response of Y input, 5 ns/div.

measurements show, these responses are all in the nanosecond range.

V. PERFORMANCE

The results given here are intended only to demonstrate some of the main features of the four-quadrant device, rather than specify the performance of a complete multiplier circuit. In most cases, a test circuit similar to that shown in Fig. 7 was used, with the scaling factor adjusted to $V_S = 1$ V, and nominal FS inputs and outputs of ± 1 V, clipping at about ± 1.5 V. The response-time tests used the 50- Ω loads presented by the inputs of a sampling scope.

Fig. 9 shows the general behavior of the device when ramp inputs are applied to the bases [Fig. 9(a)] or the emitters [Fig. 9(b)]. In each case, the other input was a dc level between -0.8 V and $+0.8$ V, in 0.2-V steps. These waveforms confirm multiplier operation, but do not reveal nonlinearity, some indication of which is provided by Fig. 9(c), which shows the feedthrough for an input of ± 1 V peak (sine wave at 1 kHz), with the other input nulled exactly. The upper trace shows the feedthrough on the Y input; as expected, it is very small, and hidden in the noise. The lower trace shows the X feedthrough, which has an amplitude of about 0.1 percent of FS, consisting mainly of second harmonic. This was the best sample of those available at the time, there being a large spread between samples, the worst having an X feedthrough of 0.6 percent of FS. Fig. 9(d) shows the step response of the Y input for a pulse of 1 V peak, with -0.8 V to $+0.8$ V on the X input. The expected variation in rise time is evident, and after factoring out the rise time of the measuring system was found to vary from 2 to 15 ns.

Fig. 10 illustrates the wide-dynamic range of this multiplier. In each case, the Y input was driven to FS with a 1 kHz sine wave, and the X input was adjusted to produce an output of

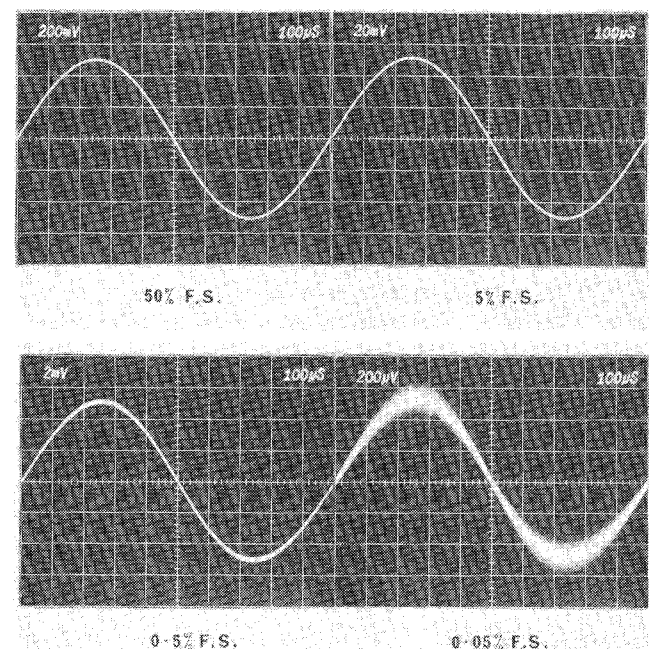


Fig. 10. Output for an FS 1-kHz sine wave input on Y and successively smaller dc inputs on X .

first 50 percent of FS, then reduced in decade steps to 0.05 percent of FS output. No serious distortion is visible over this control range of 60 dB, and the output noise level is still well below the signal in the last waveform. The signal bandwidth was 1 MHz. Using a low-noise op amp (AD504), and a Fluke 931B rms voltmeter, the noise voltage was measured for a 10 Hz to 300 kHz bandwidth, and found to be between $15 \mu\text{V}$ and $22 \mu\text{V}$ rms, corresponding to a noise spectral density of about 33 nV per root hertz, some of which is attributable to

the op amp and the rest of the interface circuitry. Over the bandwidth 10 Hz to 30 kHz this amounts to a noise voltage of only 5 μ V rms, which is 100 dB below the overload point at 1.06 V rms (± 1.5 -V peak).

Nonlinearity was originally revealed using a ramp drive to one input, derived from the oscilloscope, the vertical axis of which displayed the derivative of the output. This technique relies on a precisely linear ramp, and requires the use of a short differentiator time constant, with attendant noise problems. Also, the resulting displays are not easily translated into nonlinearity percentages. A more practical technique is to display the difference between input and output, using FS signals on X and Y , and an incremental adjustment to scaling to equalize the large-signal output to the input. This cross-plot technique showed that the Y nonlinearity was less than 0.1 percent, and limited by the interface circuitry. The X nonlinearity was much more gross, with an average value of about 1.5 percent, and a maximum value of 2.8 percent. The form of the distortion was roughly S -shaped, but varied from sample to sample.

No measurements were made of scaling accuracy and stability, since these are parameters determined to a large extent by the interface circuitry. However, it has been established that the domain location is not significantly affected by temperature variations from -20°C to $+100^\circ\text{C}$, using fixed bias supplies and DVM measurements of output.

VI. CONCLUSIONS

The carrier-domain multiplier holds promise of nearly noise-free control of attenuation or gain. Improvements in the linearity of one of the inputs is required before the device equals the performance of prevalent translinear multipliers, which in spite of the problems referred to in this paper remains the preferred technique for wide-band monolithic applications. The relatively large supply currents and voltages needed to bias a domain multiplier, and the fact that its output current appears across a finite load resistor are further ways in which it differs from a translinear device. In most other respects, however, it is comparable, and may provide a complementary technique in the long run. Variations in design, notably devices using a slender geometry and linear motion of the domain, are being explored with a view to overcoming some of the disadvantages of the reported designs.

APPENDIX I

VOLTAGE DISTRIBUTION IN A SEMICIRCULAR DEE

We wish to find the voltage around the circumference of the dee shown in Fig. 3(a) for any value of the modulation index X and the angle θ_m at which this voltage is maximum. Assuming point contacts, the result will be independent of the size of the dee, which is therefore given unit radius. The magnitude of the voltages will be proportional to the sheet resistivity ρ_B and the drive current I_B but the form of the distribution will be strictly a function of X . The analysis is simplified by considering the effect of just the input at A with $X = 0$, that is, an input of I_B . The total distribution is then found by substituting the actual values for the inputs and summing.

To provide a transformable boundary the mirror image of

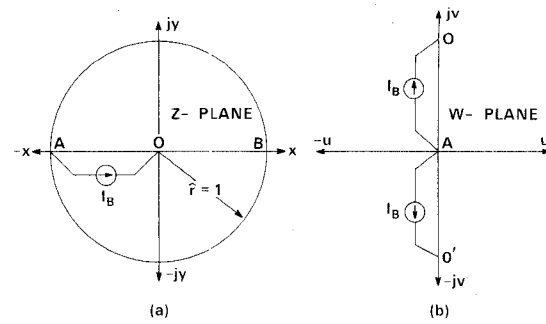


Fig. 11. Conformal transformation of circular base region for analysis.

the dee is added, and the resistivity doubled (to maintain the correct voltage scale). This unit circle is supposed to exist in the z -plane [Fig. 11(a)]. Using the transformation

$$w = j \frac{1 - z}{1 + z} \quad (15)$$

this maps to the infinite w -plane, as shown in Fig. 11(b). The origin in the z -plane moves to $w = j$, point A becomes the new origin and the point B splits outward to the circle at infinity. The boundary of the circle, across which no current flows, maps to the real axis of the w -plane, but the sources at O and A generate an asymmetric potential distribution, which would violate the original boundary condition. This problem is eliminated simply by adding an image source at O' , since the potentials are now balanced in the upper and lower half-planes. These three point sources generate similar distributions, differing only in location and magnitude and summing to

$$V(w) = \frac{2\rho_B}{2\pi} [2I_B \ln w - I_B \ln(w - j) - I_B \ln(w + j)]. \quad (16)$$

This simplifies to

$$V(w) = \frac{\rho_B I_B}{\pi} \ln \frac{w^2}{1 + w^2} \quad (17)$$

which transforms back to the z -plane as

$$V(z) = \frac{\rho_B I_B}{\pi} \ln \frac{4z}{(1 - z)^2} \quad (18)$$

which in cylindrical coordinates becomes

$$V(r, \theta) = \frac{\rho_B I_B}{\pi} \ln \frac{1 - 2r \cos \theta + r^2}{4r} \quad (19)$$

where r and θ are general points in this plane. Now applying the actual inputs to A and B and noting that $\cos(\pi/2 - \theta) = -\cos \theta$, the general solution is found to be

$$V(r, \theta, X) = \frac{\rho_B I_B}{\pi} \left[(1 - X) \ln \frac{1 - 2r \cos \theta + r^2}{4r} + (1 + X) \ln \frac{1 + 2r \cos \theta + r^2}{4r} \right]. \quad (20)$$

We are especially interested in the circumferential distribution, obtained by letting $r = 1$. This gives

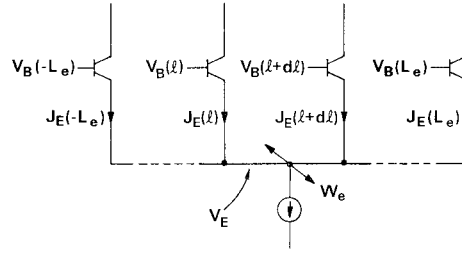


Fig. 12. Model of stripe emitter for analysis.

$$V_B(\theta, X) = \frac{2\rho_B I_B}{\pi} \left[(1-X) \ln \sin \frac{\theta}{2} + (1+X) \ln \cos \frac{\theta}{2} \right] \quad (21)$$

The maximum of this function occurs where

$$\left. \frac{\partial V_B(\theta)}{\partial \theta} \right|_{x=\text{const}} = (1-X) \cot \frac{\theta}{2} + (1+X) \tan \frac{\theta}{2} = 0 \quad (22)$$

the solution of which is

$$\theta_m = \cos^{-1} X. \quad (1)$$

The peak voltage, $V_m(X) = V_B(\theta_m, X)$ is found by putting this value of θ_m in (21):

$$V_m(X) = \frac{\rho_B I_B}{\pi} \ln [x^x (1-x)^{(1-x)}] \quad (23)$$

where

$$x = \frac{1+X}{2}. \quad (24)$$

APPENDIX II

CURRENT DENSITY IN A ONE-DIMENSIONAL DOMAIN-FORMING STRUCTURE

Fig. 12 shows the distributed structure quantized into elemental transistors for the purpose of analysis. Each elemental emitter is of width W_e and there is no transverse bias. Summation of current densities along the length axis gives the current at the emitter terminal:

$$I_E = W_e \int_{-L_e}^{L_e} J_E(l) dl. \quad (25)$$

Applying the well-known junction law

$$J_E(l) = J_s \exp \frac{V_{BE}(l)}{V_o} = J_s \exp \frac{V_B(l) - V_E}{V_o} \quad (26)$$

where

J_s is the saturation current density for the particular diffusion profiles;

$V_B(l)$ is the base voltage profile measured with respect to an arbitrary reference potential;

V_E is the voltage at the emitter terminal with respect to the same reference potential;

and

V_o is the thermal voltage kT/q .

Equation (26) can be written as

$$J_E(l) = \frac{J_s}{\exp \frac{V_E}{V_o}} \exp \frac{V_B(l)}{V_o} = J_o \exp \frac{V_B(l)}{V_o} \quad (27)$$

where J_o is a new scaling constant.

Inserting this value for $J_E(l)$ into (16) and rearranging terms:

$$J_o = \frac{I_E}{W_e \int_{-L_e}^{L_e} \exp V_B(l)/V_o dl} \quad (28)$$

APPENDIX III

APPROXIMATE SHAPE AND MAGNITUDE OF DOMAIN

An explicit solution for the current density distribution $J_E(\theta)$ can be obtained by inserting the expression for $V'_B(\theta)$ into (7) and (8). As an aid to visualizing the domain, however, it is not very informative. By using a simple approximation to $V'_B(\theta)$ in the vicinity of V_m the domain can be shown to have a Gaussian distribution, and a workable approximation for the peak current density J_o can be obtained. The proof is given for $\theta_m = 90^\circ$, but can easily be generalized to show that it holds over most of the range of θ_m .

$$\begin{aligned} V'_B(\theta) &= V_B(\theta, 0) - V_m(0) \\ &= \frac{2\rho_B I_B}{\pi} \left[\ln \sin \frac{\theta}{2} + \ln \cos \frac{\theta}{2} - \ln \sin \frac{\pi}{4} - \ln \cos \frac{\pi}{4} \right] \\ &= \frac{2\rho_B I_B}{\pi} \ln \sin \theta. \end{aligned} \quad (29)$$

Now let $V_B(\phi)$ be the voltage at angles $\pm\phi$ from 90° . Then

$$\begin{aligned} V(\phi) &= \frac{2\rho_B I_B}{\pi} \ln \sin \left(\frac{\pi}{2} \pm \phi \right) \\ &\approx -\rho_B I_B \phi^2 \quad \text{for } \phi \text{ in radians} \\ &\approx -3.05 \times 10^{-4} \rho_B I_B \phi^2 \quad \text{for } \phi \text{ in degrees.} \end{aligned} \quad (30)$$

The approximation is accurate to 1 percent up to $\phi = \pm 14^\circ$. The curves of Fig. 5 were plotted from exact data, using $2\rho_B I_B = 1$ V, which requires $I_B = 7.85$ mA for $\rho_B = 200 \Omega$ per square, and they show that the domain is contained within

this angular range, so the approximation is realistic. Therefore,

$$J_E(\phi) \approx J_o \exp \left(-\frac{\rho_B I_B \phi^2}{V_o} \right) \quad (31)$$

which is simply a Gaussian distribution in ϕ . By the use of (8) the value of J_o is found to be

$$J_o \approx \frac{I_E}{\pi R_E W_E} \sqrt{\frac{2 \rho_B I_B}{V_o}}. \quad (32)$$

Inserting typical values of $I_E = 1$ mA, $R_E = 100 \mu\text{m}$, $W_E = 10 \mu\text{m}$, $2 \rho_B I_B = 1$ V, $V_o = 26$ mV, the approximate value of J_o works out as $197 \text{ A} \cdot \text{cm}^{-2}$.

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